This is the reference manual of LibTMCG.
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1 Introduction

‘LibTMCG’ is a C++ library for creating secure and verifiable online card games. The library contains a sort of useful classes, algorithms, and high-level protocols to support an application programmer in writing such software. The most remarkable feature is the absence of a trusted third party (TTP), i.e., neither a central game server nor trusted hardware components are necessary. Thus, with the present library there is no need for an independent referee, because the applied protocols provide a basic level of confidentiality and verifiability by itself. Consequently, the library is well-suited for peer-to-peer (P2P) environments where no TTP is available. Of course, we cannot avoid that malicious players share information about their private cards, but the protocols ensure that the shuffle of the deck is performed randomly (presumed that at least one player is honest) and thus the cards will be distributed uniformly among the players. Further, no coalition can learn the private cards of a player against his will (except for trivial conclusions). The corresponding cryptographic problem, actually called “Mental Poker”, has been studied since 1979 (Shamir, Rivest, and Adleman) by many authors. LibTMCG provides the first practical implementation of such sophisticated cryptographic protocols.

The security and the verifiability rely on advanced cryptographic techniques—the so-called zero-knowledge proofs. Using these ‘building blocks’ the high-level protocols minimize the effect of coalitions and preserve the confidentiality of the players’ strategy, i.e., the players are not required to reveal their cards at the end of the game in order to show that they did not cheat. This important property is often required in card games like Poker, where not all cards are opened during the play and the applied individual strategy must be kept secret.

LibTMCG is Free Software according to the definition of the Free Software Foundation. The source code is released under the GNU General Public License Version 2.

1.1 Further Reading

The cryptographic background and a detailed discussion of the implementation issues are beyond the scope of this manual. The interested reader is referred to the following scientific papers:

http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.29.6679

http://dx.doi.org/10.1007/978-3-540-40974-8_29

http://eprint.iacr.org/2005/246

http://dx.doi.org/10.1007/s00145-010-9067-9

http://dx.doi.org/10.1007/978-3-642-00468-1_22

Chapter 1: Introduction

1.2 Getting Started

This manual describes the application programming interface of LibTMCG. All relevant data types, public classes and security parameters are explained. The reader should have an advanced knowledge in applied cryptography and C++ programming. Reference is made at this point to the famous Handbook of Applied Cryptography for a brief introduction on the first topic. For the underlying communication model and some broadcast primitives the outstanding textbook Introduction to Reliable and Secure Distributed Programming and the corresponding exercises are recommended.
This document follows, in style and rarely in phrasing, the Libgcrypt Reference Manual. Thus don’t be surprised, if you recognize some obvious analogies.

1.3 Preliminaries

The most card games are played with a regular card deck, i.e., cards where the pattern on the front side (face) determines the card type (e.g. the King of Spades $♠$, the Seven of Hearts $♥$, the Ace of Club $♣$, or the Jack of Diamonds $♦$) and where the back sides (face down) of all cards are indistinguishable. Only such ‘regular’ card decks are supported by LibTMCG and the provided card encoding schemes.

1.3.1 Terminology

The following list defines some common terms that are subsequently used in the manual.

- **Player**: A player is an active participant in an electronic card game.
- **Observer**: An observer is an passive party who watches the game.
- **Card**: The term card means the electronic representation of a playing card.
- **Card Type**: The card type is a nonnegative integer which corresponds to the pattern on the picture side of a real playing card. We assume here that such a natural encoding always exists.
- **Masking**: Masking is a process which aim is to transform the card representation such that the input card and the result cannot be linked (except for trivial conclusions). Roughly speaking, masking is the (re-)encryption of a card representation such that the original card type is preserved.
- **Card Secret**: The card secret contains all random values used in a masking operation. These values must be kept secret until the card is publicly revealed. Otherwise the corresponding output of the masking transformation is linkable and other players may learn the card type.
- **Open Card**: An open card is a card whose type can be easily determined by all players and usually by observers as well.
- **Masked Card**: A masked card (also known as face-down card) is a card whose type is unknown to a subset of players. It can be only revealed, if all players cooperate in a common computation of the type.
- **Private Card**: A private card is a card whose type is only known to its owner. As long as the owner does not cooperate the type of the private card stays hidden to all other players (except for trivial conclusions).
- **Stack**: A stack is a not necessarily disjoint subset of the whole card deck.
- **Prover and Verifier**: The prover is a player who shows some property to another party called verifier. For example, he wants to show that a masking operation was performed correctly, i.e., the card type is preserved by the transformation.

1.3.2 Security

“Mental Poker” solutions cannot prevent that malicious players exchange private information, for example, by telephone or Internet chat. Cryptographic protocols can only minimize the effect of such colluding parties and should try to protect the confidentiality for honest players. But even this small protection often relies on number-theoretical assumptions which are only believed to be true, i.e., problems like factoring products of large primes or computing discrete logarithms are only believed to be hard. That means, strict mathematical proofs\(^\dagger\) for the hardness of these problems are not known, and it is not very likely that such proofs will ever be found. However,

\(^\dagger\) For instance, a “tight reduction” to a known hard problem in the sense of complexity theory.
almost all public key cryptosystems rely on such assumptions and therefore you should not care
about this issue, as long as reasonable security parameters are chosen and practical quantum
computers are out of range.

LibTMCG was originally designed to provide security in the “honest-but-curious” (aka “semi-
honest” or passive) adversary model. That means, all participants follow the protocol instruc-
tions properly but they may gather information and share them within a coalition to obtain an
advantage in the game. Thus we are basically not concerned with robustness and availability
issues which are hard to solve in almost asynchronous environments like the Internet. However,
the most operations are verifiable such that cheating can be detected. To obtain this verifica-
bility, the protocols deploy so-called zero-knowledge proofs which yield no further knowledge
but the validity of a statement. The soundness error of these proofs is bounded by a fixed
security parameter $\kappa$. Depending on your application scenario this parameter should be chosen
such that there is a reasonable tradeoff between the cheating probability (which is less or equal
than $2^{-\kappa}$) and the produced computational and communication complexity. LibTMCG also
uses so-called zero-knowledge proofs of knowledge due to Bellare and Goldreich (see On defining
proofs of knowledge, Advances in Cryptology – Proceedings of CRYPTO’92, 1992), however, for
convenience we will not further distinguish between these building blocks. Finally, some of the
protocols (e.g. the efficient shuffle argument by Groth) are only zero-knowledge with respect to
a so-called “honest verifier” who follows all protocol instructions faithfully. Since version 1.2.0
of LibTMCG we use a two-party version of a distributed coin flipping protocol by Jarecki and
Lysyanskaya [JL00] to protect against malicious verifiers in that case.

Unfortunately, in practice there is another substantial problem with the detection of cheaters.
It requires that an authenticated broadcast channel has been set up, where all players have
read/write access. There exist protocols (so-called “reliable broadcast” or even “atomic broad-
cast”) for creating such a channel, however, only under the additional condition that the number
of parties $t$ who act faulty or even malicious (so-called “Byzantine adversary”) is reasonable
small. In a full asynchronous environment like the Internet resilience is achievable for $t < n/3$
only, where $n$ denotes the total number of parties in the protocol. LibTMCG provides a well-
known protocol due to Bracha (see An asynchronous $(n - 1)/3$-resilient consensus protocol,
optimized variant by Cachin, Kursawe, Petzold, and Shoup [CKPS01]. Please note that in most
cases the application programmer must decide, where the use of a broadcast channel is nece-
sary and appropriate. Thus, without reliable broadcast you should take into account that not
necessarily the player acting as prover is the source of evil, if a verification procedure fails. This
level of uncertainty is the main reason for our still limited adversary model.

Note that it is not known, whether the used protocols retain their zero-knowledge property,
if they are composed and executed in a concurrent setting. Thus the application programmer
should be careful and avoid parallel protocol sessions. It is an open research project to create
a protocol suite whose security can be proven in the UC-framework of Canetti (see Universally
Composable Security: A New Paradigm for Cryptographic Protocols, Cryptology ePrint Archive:
Report 2000/067) or even more elaborated UC-frameworks (see e.g. Dennis Hofheinz and Victor
2011/303). Furthermore, the protocols should employ concurrent zero-knowledge proofs (see
Cynthia Dwork, Moni Naor, and Amit Sahai: Concurrent Zero-Knowledge, Journal of the ACM

Please also note, that in some protocols the Fiat-Shamir heuristic [FS87] is used to turn inter-
active special honest verifier zero-knowledge arguments resp. proofs into non-interactive versions
in the random oracle model. However, there are some theoretical (see e.g. Nir Bitansky, Dana
Dachman-Soled, Sanjam Garg, Abhishek Jain, Yael Tauman Kalai, Adriana Lopez-Alt, and
Daniel Wichs: Why ‘Fiat-Shamir for Proofs’ Lacks a Proof, TCC 2013, LNCS 7785, 2013) and
practical (see David Bernhard, Olivier Pereira, Bogdan Warinschi: How Not to Prove Yourself:
Pitfalls of the Fiat-Shamir Heuristic and Applications to Helios, ASIACRYPT 2012, LNCS 7658, 2012) concerns that show the insecurity of Fiat-Shamir heuristic w.r.t. the soundness of the argument resp. proof. That means, if deterministic hash functions are used as public coin [BR93], then the random oracle assumption obviously does not hold and therefore a malicious prover can manipulate the challenges in order to cheat and thus violates the soundness property. On the other hand, the Fiat-Shamir heuristic, and in general the non-interactivness of the transformed protocols, protect against a malicious verifier. Thus it is another important measure to deal with the limitation of honest verifier zero-knowledge proofs resp. arguments of knowledge without loosing their efficiency. However, non-interactive protocols are necessarily malleable (when used without unique identifiers), and the cheating verifier can generate a convincing proof of knowledge by copying one sent by the prover in a previous iteration of the protocol. This issue must be adressed by the application programmer, for example, by using fresh randomness in each card or stack operation which should be verifiable.

LibTMCG was carefully implemented with respect to timing attacks (see Paul C. Kocher: Cryptanalysis of Diffie-Hellman, RSA, DSS, and other cryptosystems using timing attacks, CRYPTO ’95, LNCS 963, 1995). Therefore we loose some efficiency, e.g., during modular exponentiations. However, it is strongly recommended to leave the timing attack protection turned on, unless you know exactly where it is really not needed.

Security Advice: We have implemented all cryptographic primitives according to the cited research papers and to the best of our knowledge. However, we can not eliminate any possibility of contained flaws or bugs, because the implementation of such complex protocols is always an error-prone process. Moreover, the scientific results are sometimes controversial or even wrong. Thus we encourage readers with advanced cryptographic background to review given references and the source code of LibTMCG. Please report any complaint or correction proposal!

1.3.3 Communication
Most cryptographic protocols are designed for a synchronous communication model, i.e., there is a known upper bound on message transmission delays. That means, the time period between the point at which a protocol message is sent and the point at which the message is delivered is smaller than this bound. Additionally, often the assumption is made that the computation proceeds in synchronized rounds and that the parties are connected by a complete network of private (i.e. untappable and authenticated) point-to-point channels.

There is an important distinction between fully synchronous and partially synchronous communication model with respect to coverage and the resulting adversarial power. However, a detailed discussion of such issues is beyond the scope of this manual. The reader is referred to the famous textbook Introduction to Reliable and Secure Distributed Programming for an introduction and discussion on that topic.

1.4 Preparation
LibTMCG depends on three other basic libraries. Therefore you will need the corresponding development files to build LibTMCG and your application properly. The following list gives a short exposition of the used features and specifies the required versions:

- GNU Multiple Precision Arithmetic Library (libgmp), Version ≥ 4.2.0
  The library provides a powerful framework for performing arbitrary precision arithmetic on integers. Further reasons for choosing this dependency are the license compatibility, the portability, the vital maintenance, and of course, the reasonable performance.

- GNU Crypto Library (libgcrypt), Version ≥ 1.6.0
  The library provides some basic cryptographic algorithms (e.g. SHA-256, AES256, ElGamal, DSA, RSA) and an easily accessible interface for cryptographically strong pseudo
random numbers. If a version $\geq 1.7.0$ is found, then also the hash function SHA-3 will be used.

- GNU Privacy Guard Error Code Library (libgpg-error), Version $\geq 1.12$

This library defines common error values, e.g., returned by the GNU Crypto Library.

We suppose that the reader is familiar with these libraries because their correct installation, configuration, and usage is crucial to the security of the entire application.

### 1.5 Header Files and Name Spaces

The interface definitions of classes, data types, and security parameters\(^2\) are provided by the central header file `libTMCG.hh`. You have to include this file in all of your sources, either directly or through some other included file. Thus often you will simply write:

```c
#include <libTMCG.hh>
```

There are no uniform C++ name spaces for the most parts of the library. Some classes and data types have the common prefix `TMCG_*` resp. `VTMF_*` while others are composed of the author names and an abbreviation of the title from the related research paper. Further there are some function names and types that are prepended by `tmcg_*`. Those interfaces should be used with care, because later they may be removed or replaced.

### 1.6 Building Sources

If you want to compile a source file including the `libTMCG.hh` header, you must make sure that the compiler can find it in the directory hierarchy. This is achieved by adding the path of the corresponding directory to the compilers include file search path.

However, the path to the include file has been determined at the time the source is configured. To solve this problem, LibTMCG ships with a small helper program `libTMCG-config` that knows the path to the include file and a few other configuration options. The options that need to be added to the compiler invocation are output by the `--cflags` option to `libTMCG-config`.

The following example shows how it can be used at the command line:

```bash
g++ -c foo.cc 'libTMCG-config --cflags'
```

Adding the output of `libTMCG-config --cflags` to the compilers command line will ensure that the compiler can find the LibTMCG header file.

A similar problem occurs when linking your program with LibTMCG. Again, the compiler has to find the library files. Therefore the correct installation path has to be added to the library search path. To achieve this, the option `--libs` of `libTMCG-config` can be used. For convenience, this option also outputs all other stuff (e.g. required third-party libraries) that is required to link your program with LibTMCG (in particular, the `-lTMCG` option).

The example shows how to link `foo.o` with LibTMCG to a program called `foo`:

```bash
g++ -o foo foo.o 'libTMCG-config --libs'
```

Of course, you can also combine both examples to a single command by calling the shell script `libTMCG-config` with both options:

```bash
g++ -o foo foo.o 'libTMCG-config --cflags --libs'
```

\(^2\) Some security parameters are fixed at compile time of LibTMCG. Please don’t change anything unless you know exactly what you are doing! Beside the apparent security concerns you will probably break the compatibility with other applications using LibTMCG.
1.6.1 Building Sources Using GNU Automake

You can use GNU Automake to obtain automatically generated Makefiles. If you do so then you do not have to care about finding and invoking the `libTMCG-config` script at all. LibTMCG provides an Automake extension that does all the stupid work for you.

\[
\text{AM_PATH_LIBTMCG} \left( \text{[minimum-version], [action-if-found], [action-if-not-found]} \right) \]

Check whether LibTMCG (at least version minimum-version, if given) exists on the host system. If it is found, execute action-if-found, otherwise do action-if-not-found. Additionally, the macro defines \text{LIBTMCG_CFLAGS} to the flags needed for compilation in order to find the necessary header files, and \text{LIBTMCG_LIBS} to the corresponding linker flags.

You can use the defined variables in your \texttt{Makefile.am} as follows:

\[
\text{AM_CPPFLAGS} = \$(\text{LIBTMCG_CFLAGS}) \\
\text{LDADD} = \$(\text{LIBTMCG_LIBS})
\]

1.7 Initializing the Library

The first step is the initialization of LibTMCG. The following function must be invoked early in your program, i.e., before you make use of any other capability of LibTMCG.

\[
\text{bool init_libTMCG} \left( \text{const bool force_secmem} = \text{false, const bool gmp_secmem} = \text{false, const size_t max_secmem} = 32768} \right) \]

The function checks whether the installed third-party libraries match their required versions. Further it initializes them and returns true, if everything was sound. Otherwise false is returned and an appropriate error message is sent to \texttt{std::cerr}.

The three optional arguments define the behaviour concerning the allocation of secure memory (i.e., memory that is not paged out to disk and that is overwritten by zeros before released) from libgcrypt. By default no secure memory is used. If force_secmem is true, than those parts of LibTMCG that use the GNU Crypto Library will allocate and use secure memory for private keys or other secrets. However, the most classes, algorithms, and protocols of LibTMCG does not respect this option yet, because they store their secrets with the GNU Multiple Precision Arithmetic Library. With the second option gmp_secmem the default memory allocator of this library is replaced to use secure memory. Unfortunately, there is no way to specify whether a big integer needs secure memory and thus all memory is allocated in this fashion. This may lead to out of memory aborts, because the allocated secure memory is limited (currently 32kB). The limit of libgcrypt can be adjusted by the third parameter max_secmem, however, probably there are restrictions of the operating system (cf. RLIMIT_MEMLOCK).

Additionally, the function \text{version_libTMCG} returns a string containing the version number of the library in a common format. It is strongly recommended to check, whether the installed version matches your requirements.

\[
\text{const std::string version_libTMCG} () \]

This function returns the version of the library in the format major.minor.revision.

Last but not least, there is a function \text{identifier_libTMCG} which returns an identifier of LibTMCG including the version, copyright mark and license.

\[
\text{const std::string identifier_libTMCG} () \]

This function returns an identifier of the library.
2 Application Programming Interface

Now we start with a description of some important global symbols and structures.

2.1 Preprocessor Defined Global Symbols

Please note that the following macros are fixed at compile time of LibTMCG and cannot be changed by your application. They are only provided here for informational purposes.

**TMCG_MR_ITERATIONS** [Macro]
Defines the number of iterations for the Miller-Rabin primality test. The default value is 64 which implies a soundness error probability \( \leq 4^{-64} \).

**TMCG_MAX_ZNP_ITERATIONS** [Macro]
Defines the maximum number of iterations for the prover in cut-and-choose style zero-knowledge protocols of Schindelhauer’s toolbox. The default value is 80 which limits the soundness error probability to \( \geq 2^{-80} \), however, it protects against some obvious denial-of-service attacks from a malicious verifier.

**TMCG_GROTH_L_E** [Macro]
Defines the security parameter \( \ell_e \) of Groth’s (interactive) shuffle argument [Gr05]. The default value is 80 which implies a soundness error probability \( \leq 2^{-80} \). For the intended purposes of LibTMCG this seems to be reasonable.

**TMCG_DDH_SIZE** [Macro]
Defines the security parameter (finite field size in bit) of the group \( G \) which is used by the card encoding scheme of Barnett and Smart [BS03]. The underlying assumptions are DDH, CDH, and DLOG. The default value is 2048.

**TMCG_DLSE_SIZE** [Macro]
Defines the security parameter (subgroup size in bit) of the group \( G \) which is used by the card encoding scheme of Barnett and Smart [BS03]. The underlying assumptions are DLSE (related to DDH) and DLOG. The default value is 256.

**TMCG_AIO_HIDE_SIZE** [Macro]
Defines the security parameter for hiding the length of integers in derived classes from aiounicast. The default value is 256.

**TMCG_GCRY_MD_ALGO** [Macro]
Defines the main message digest algorithm (i.e. hash function \( h() \)) for digital signatures with PRab [BR96] and mask generation for Rabin encryption with SEAP [Bo01] in TMCG_SecretKey. This algorithm is also used for the construction of a special hash function \( g() \), which is needed for the Fiat-Shamir heuristic [FS87]. Recently we switched\(^1\) to the hash function SHA-256 (default value \texttt{GCRY_MD_SHA256}) for improved collision resistance. Thus we gain a security level of approximately \( 2^{128} \), assuming that a birthday-attack is the best known attack against this message digest.

Please note that the security of the non-interactive zero-knowledge proofs resp. arguments (NIZK) is proved in the so-called random oracle model (ROM), i.e., we suppose that the instantiated hash function \( g() \) behaves like an ideal random function (which obviously cannot hold in a real world scenario with deterministic computations). However, this assumption seems to be reasonable, if the underlying hash function is collision-resistant and if it is carefully implemented with respect to other instantiations [BR93].

---

\(^1\) In former versions of LibTMCG the default value of this symbol was \texttt{GCRY_MD_RMD160}, i.e. the hash algorithm RIPemd-160 (see Dobbertin, Bosselaers, Preneel: \textit{RIPEMD-160, a strengthened version of RIPEMD, 1996}), which is a function that has only an output length of 160 bit.

\(^2\) This is also a constant defined by the GNU Crypto Library.
**TMCG_GCRY_MAC_ALGO** [Macro]
Defines the message authentication algorithm for authenticated channels established by the class `aiounicast`. The default value is `GCRY_MAC_HMAC_SHA256`, i.e. the HMAC based scheme with hashing algorithm SHA-256.

**TMCG_GCRY_ENC_ALGO** [Macro]
Defines the symmetric encryption algorithm (sometimes also called cipher) for private channels established by the class `aiounicast`. The default value is `GCRY_CIPHER_AES256`, i.e. the cipher AES256, which is used by LibTMCG in CFB (Cipher Feedback) mode.

**TMCG_KEYID_SIZE** [Macro]
Defines the length (in characters w.r.t. `TMCG_MPZ_IO_BASE`) for the distinctive suffix of the unique TMCG key identifier. The default value is 8 which spans a reasonable name space for at least $2^{20}$ different TMCG keys (see `TMCG_PublicKey`). However, sometimes it is required to use even smaller sizes due to artificial protocol restrictions (e.g. the IRC nickname is sometimes restricted to 9 characters).

Each key identifier starts with the string "ID" followed by the decimal encoded value of `TMCG_KEYID_SIZE` and the appended carret symbol "^". The final suffix contains `TMCG_KEYID_SIZE` alphanumerical characters from the self signature of TMCG key. This signature has enough entropy included to be used as unique key identifier.

**TMCG_KEY_NIZK_STAGE1** [Macro]
Defines the security parameter (number of iterations) of the NIZK proof [GMR98] (stage 1) which convince all verifiers that the TMCG key was correctly generated. The default value is 16 which implies a soundness error probability $\leq d^{-16}$, where $d = \gcd(m, \phi(m))$ and $m$ is part of the public key. This parameter is only relevant for the card encoding scheme of Schindelhauer, where the key has a very special format.

**TMCG_KEY_NIZK_STAGE2** [Macro]
Defines the security parameter (number of iterations) of the NIZK proof [GMR98] (stage 2) which convince all verifiers that the TMCG key was correctly generated. The default value is 128 which implies a soundness error probability $\leq 2^{-128}$. This parameter is only relevant for the card encoding scheme of Schindelhauer.

**TMCG_KEY_NIZK_STAGE3** [Macro]
Defines the security parameter (number of iterations) of the NIZK proof [Sc98] (stage 3) which convince all verifiers that the TMCG key was correctly generated. The default value is 128 which implies a soundness error probability $\leq 2^{-128}$. This parameter is only relevant for the card encoding scheme of Schindelhauer.

**TMCG_LIBGCRYPT_VERSION** [Macro]
Defines the required minimum version number of the GNU Crypto Library. The default value is "1.6.0". During the initialization of LibTMCG (see `init_libTMCG`) it is checked, whether the version number of the linked shared object fulfills this condition.

**TMCG_LIBGMP_VERSION** [Macro]
Defines the required minimum version number of the GNU Multiple Precision Arithmetic Library. The default value is "4.2.0". During the initialization of LibTMCG (see `init_libTMCG`) it is checked, whether the version number provided by the header file `gmp.h` and used at compile time of LibTMCG fulfills this condition.

---

3 This is also a constant defined by the GNU Crypto Library.
4 This is also a constant defined by the GNU Crypto Library.
Chapter 2: Application Programming Interface

**TMCG\_MAX\_CARDS**
Defines the maximum number of stackable cards. The default value is 1024.

**TMCG\_MAX\_PLAYERS**
Defines the maximum number of players. The default value is 32. This parameter is only relevant for the card encoding scheme of Schindelhauer.

**TMCG\_MAX\_TYPEBITS**
Defines the maximum number of bits to represent the card type in the scheme of Schindelhauer. On the other hand, this value determines the maximum size of the message space in the scheme of Barnett and Smart. The default value is 10 which implies that 1024 different card types are possible. For each type some memory will be allocated, thus this value should modified very carefully.

**TMCG\_MPZ\_IO\_BASE**
Defines the input and output base of the `std::iostream` operators `<<` and `>>` which is used to encode large integers (`mpz_t`). The former value was 36 which was some years ago the largest base supported by the GNU Multiple Precision Arithmetic Library. Since version 1.2.0 of LibTMCG the new default value is 62.

**TMCG\_PRAB\_K0**
Defines the security parameter $k_0$ (in characters) of the PRab scheme [BR96]. The default value is 20 which implies a security level around $2^{80}$.

**TMCG\_QRA\_SIZE**
Defines the security parameter (size of the modulus $m = p \cdot q$ in bit) of the TMCG key. The underlying assumptions are QRA and FACTOR. The default value is 2048. This parameter is only relevant for TMCG keys and Schindelhauer’s encoding scheme.

**TMCG\_SAEP\_S0**
Defines the security parameter $s_0$ (in characters) of the Rabin-SAEP scheme [Bo01]. The default value is 20 which implies a security level around $2^{80}$ against CCA (chosen-ciphertext attack).

**TMCG\_HASH\_COMMITMENT**
Defines whether shortened commitments are used in the shuffle verification procedure of Schindelhauer [Sc98]. The default value is `true`, because this will decrease the communication complexity significantly. However, as an immediate consequence the soundness property is violated, if the hash function `TMCG\_GCRY\_MD\_ALGO` is broken.

**TMCG\_MAX\_FPOWM\_T**
Defines the maximum size of admissible exponents (in bit) used by our fast exponentiation procedures. The default value is 2048. Note that this parameter has a strong influence on the amount of memory allocated by LibTMCG since it determines the size of the precomputed tables. However, it should be at least greater or equal than `TMCG\_DDH\_SIZE` and `TMCG\_QRA\_SIZE` in order to support the possible exponents of common finite field sizes.

**TMCG\_MAX\_FPOWM\_N**
Define the maximum number of different bases for doing the above precomputation. This value is a trade-off between fast exponentiation for all possible bases and memory allocation. Currently it is only relevant for the generators $g_1, \ldots, g_n$ in Groth’s variant of Pedersen commitment scheme (see Section 2.2.3.4 [GrothVSSHE], page 34). The default value is 256.

**TMCG\_MAX\_SSRANDOMM\_CACHE**
Define the maximum size of the cache for function `mpz\_ssrandomm`. The cache must be proper initialized and is useful in interactive protocols, where entropy is limited and a lot of very
secure randomness is required immediately. Thus some values should be acquired and cached before the protocol starts. The default value is 256.

2.2 Basic Structures

This section describes all public data types, communication interfaces, and classes of high-level protocols that are necessary to create a secure card game. Private methods and only internally used members are not explained.

2.2.1 Data Types

LibTMCG provides several data structures for cards, stacks, and cryptographic keys.

2.2.1.1 Encoding Schemes for Cards

There exist two different encoding schemes that can be used for the digital representation of playing cards. In the scheme of Schindelhauer [Sc98] the type of a card is shared among the players through bit-wise representation by quadratic (non-)residues. Thus the security relies on the well-known QRA (Quadratic Residuosity Assumption). Unfortunately, the size of a card grows linearly in the number of players and logarithmically in the number of card types. Recently the much more efficient solution of Barnett and Smart [BS03] has been implemented. This encoding works on a cyclic group of prime order and requires that the DDH (Decisional Diffie-Hellman Assumption) holds there.

For both schemes LibTMCG provides a structure whose name contains the suffix Card. This data type is used to represent an open or even a masked card. Further, there is a corresponding structure whose name contains the suffix CardSecret. This data type is used to represent the secret values involved in a card masking operation.

Because of the reduced computational and communication complexity (see [St05] for more details) the usage of the second card encoding scheme, i.e. VTMF_Card and VTMF_CardSecret, is highly recommended.

**TMCG_Card**

This struct represents a card in the encoding scheme of Schindelhauer [Sc98]. The type of the card is shared among the players by quadratic residues and non-residues, respectively. Thus the security relies on the Quadratic Residuosity Assumption.

std::vector< std::vector<MP_INT> > z

This \( k \times w \)-matrix encodes the type of the corresponding card in a shared way. For each of the \( k \) players there is a separate row and for each of the \( w \) bits in the binary representation of the type there is a column. The elements are numbers from the group \( \mathbb{Z}_{m_i} \), where \( m_i \) is the public modulus of the \( i \)th player.

**TMCG_Card ()**

This default constructor initializes the card with an empty \( 1 \times 1 \)-matrix. Later the method TMCG_Card::resize can be used to enlarge the card representation.

**TMCG_Card (size_t k, size_t w)**

This constructor initializes the card with an empty \( k \times w \)-matrix. The parameter \( k \) is the number of players and \( w \) is the maximum number of bits used by the binary representation of the card type.

**TMCG_Card (const TMCG_Card& that)**

This is a simple copy-constructor and that is the card to be copied.

**TMCG_Card& = (const TMCG_Card& that)**

This is a simple assignment-operator and that is the card to be assigned.
bool == (const TMCG_Card& that)
  This operator tests two card representations for equality.

bool != (const TMCG_Card& that)
  This operator tests two card representations for inequality.

void resize (size_t k, size_t w)
  This method resizes the representation of the card. The current content of the member z
  will be released and a new $k \times w$-matrix is created. The parameter $k$ is the number of
  players and $w$ is the maximum number of bits used by the binary representation of the
  card type.

bool import (std::string s)
  This method imports the content of the member $z$ from the correctly formatted input
  string $s$. It returns true, if the import was successful.

~TMCG_Card ()
  This destructor releases all occupied resources.

std::ostream& << (std::ostream& out, const TMCG_Card& card)
  This operator exports the content of the member $z$ (of the given TMCG_Card card) to the
  output stream $out$.

std::istream& >> (std::istream& in, TMCG_Card& card)
  This operator imports the content of the member $z$ (of the given TMCG_Card card) from the
  input stream $in$. The data has to be delimited by a newline character. The failbit of the
  stream is set, if any parse error occurred.

TMCG_CardSecret
  This struct represents the secret used for a card masking operation in the original encoding
  scheme of Schindelhauer [Sc98].

std::vector< std::vector<MP_INT> > r
  This $k \times w$-matrix encodes the first part of the secret. For each of the $k$ players there is a
  separate row and for each of the $w$ bits in the binary representation of the corresponding
  card type there is a column. The elements are numbers from the group $Z_{m_i}^*$ where $m_i$ is
  the public modulus of the $i$th player.

std::vector< std::vector<MP_INT> > b
  This $k \times w$-matrix encodes the second part of the secret. For each of the $k$ players there is a
  separate row and for each of the $w$ bits in the binary representation of the corresponding
  card type there is a column. The elements are simply numbers from \{0, 1\}.

TMCG_CardSecret ()
  This default constructor initializes both members with an empty $1 \times 1$-matrix. Later the
  method TMCG_CardSecret::resize can be used to enlarge the card representation.

TMCG_CardSecret (size_t k, size_t w)
  This constructor initializes both members with an empty $k \times w$-matrix. The parameter $k$
  is the number of players and $w$ is the maximum number of bits used by the binary
  representation of the corresponding card type.

TMCG_CardSecret (const TMCG_CardSecret& that)
  This is a simple copy-constructor and that is the secret to be copied.
TMCG_CardSecret& = (const TMCG_CardSecret& that)  
This is a simple assignment-operator and that is the secret to be assigned.

void resize (size_t k, size_t w)  
This method resizes the representation of the secret. The current content of the members \( r \) and \( b \) will be released and new \( k \times w \)-matrices are created. The parameter \( k \) is the number of players and \( w \) is the maximum number of bits used by the binary representation of the corresponding card type.

bool import (std::string s)  
This method imports the content of the members \( r \) and \( b \) from the correctly formatted input string \( s \). It returns true, if the import was successful.

~TMCG_CardSecret ()  
This destructor releases all occupied resources.

std::ostream& << (std::ostream& out, const TMCG_CardSecret& cardsecret)  
This operator exports the content of the members \( r \) and \( b \) (of the given TMCG_CardSecret cardsecret) to the output stream out.

std::istream& >> (std::istream& in, TMCG_CardSecret& cardsecret)  
This operator imports the content of the members \( r \) and \( b \) (of the given TMCG_CardSecret cardsecret) from the input stream in. The data has to be delimited by a newline character. The failbit of the stream is set, if any parse error occurred.

VTMF_Card  
This struct represents a card in the encoding scheme of Barnett and Smart [BS03]. Here we use the discrete logarithm based instantiation of their general cryptographic primitive VTMF (Verifiable k-out-of-k Threshold Masking Function). The security relies on the DDH assumption in the underlying abelian group \( G \).

mpz_t c_1  
This is the first part of the encrypted card type. It is an element from the underlying group \( G \).

mpz_t c_2  
This is the second part of the encrypted card type. It is also an element from the underlying group \( G \).

VTMF_Card ()  
This default constructor initializes an empty card where the members \( c_1 \) and \( c_2 \) are set to zero.

VTMF_Card (const VTMF_Card& that)  
This is a simple copy-constructor and that is the card to be copied.

VTMF_Card& = (const VTMF_Card& that)  
This is a simple assignment-operator and that is the card to be assigned.

bool == (const VTMF_Card& that)  
This operator tests two card representations for equality.

bool != (const VTMF_Card& that)  
This operator tests two card representations for inequality.
bool import (std::string s) [Method on VTMF_Card]
This method imports the content of the members \(c_1\) and \(c_2\) from a correctly formatted input string \(s\). It returns \texttt{true}, if the import was successful.

~VTMF_Card () [Destructor on VTMF_Card]
This destructor releases all occupied resources.

std::ostream& << (std::ostream& out, const VTMF_Card& card) [Operator on VTMF_Card]
This operator exports the content of the members \(c_1\) and \(c_2\) (of the given VTMF_Card card) to the output stream \(out\).

std::istream& >> (std::istream& in, VTMF_Card& card) [Operator on VTMF_Card]
This operator imports the content of the members \(c_1\) and \(c_2\) (of the given VTMF_Card card) from the input stream \(in\). The data has to be delimited by a newline character. The \texttt{failbit} of the stream is set, if any parse error occurred.

VTMF_CardSecret [Data type]
This \texttt{struct} represents the secrets used in the card masking operation by the encoding scheme of Barnett and Smart [BS03].

\[ \texttt{mpz_t } r \] [Member of VTMF_CardSecret]
This member is the exponent (randomizer) used in the masking operation. It should be chosen uniformly and randomly from \(\mathbb{Z}_q\) where \(q\) is the order of the finite abelian group \(G\) for which the DDH assumption holds.

According to the results of Koshiba and Kurosawa (see \textit{Short Exponent Diffie-Hellman Problems}, PKC 2004, LNCS 2947) the length of this exponent can be shorten to a more efficient size (e.g. 160 bit), if the corresponding generator of \(G\) is adjusted as well. Under the additional DLSE (Discrete Logarithm with Short Exponents) assumption the DDH problem in \(G\) seems to be still hard. By such an optimization trick we gain a great performance advantage for almost all modular exponentiations that are computed during the masking operation, if the VTMF primitive was instantiated by the later explained class \texttt{BarnettSmartVTMF_dlog_GroupQR}. Furthermore, the size of the card secret is substantially reduced which results in an improved communication complexity.

VTMF_CardSecret () [Constructor on VTMF_CardSecret]
This default constructor initializes the secret with an empty member \(r\).

VTMF_CardSecret (const VTMF_CardSecret& that) [Constructor on VTMF_CardSecret]
This is a simple copy-constructor and \(that\) is the secret to be copied.

VTMF_CardSecret& = (const VTMF_CardSecret& that) [Operator on VTMF_CardSecret]
This is a simple assignment-operator and \(that\) is the secret to be assigned.

bool import (std::string s) [Method on VTMF_CardSecret]
This method imports the content of the member \(r\) from the correctly formatted input string \(s\). It returns \texttt{true}, if the import was successful.

~VTMF_CardSecret () [Destructor on VTMF_CardSecret]
This destructor releases all occupied resources.

std::ostream& << (std::ostream& out, const VTMF_CardSecret& cardsecret) [Operator on VTMF_CardSecret]
This operator exports the content of the member \(r\) (of the given VTMF_CardSecret cardsecret) to the output stream \(out\).
std::istream & >> (std::istream & in, VTMF_CardSecret & cardsecret) [Operator on VTMF_CardSecret]
This operator imports the content of the member r (of the given VTMF_CardSecret cardsecret) from the input stream in. The data has to be delimited by a newline character. The failbit of the stream is set, if any parse error occurred.

2.2.1.2 Stacks
All of the following data types are generic containers that can be instantiated as C++ templates with the former explained Card and CardSecret data types, respectively. Note the maximum number of stackable data is upper-bounded by TMCG_MAX_CARDS. There is no error reported, if this limit is exceeded.

TMCG_Stack<
CardType>
This struct is a simple container for cards of the specified CardType. Currently, the elements can be either of type TMCG_Card or VTMF_Card depending on which kind of encoding scheme is used. The TMCG_Stack structure is mainly used to represent a stack of masked cards, i.e., playing cards that are stacked in a face-down manner. It can be either a public stack where all participants have access to or even a private stack, e.g. the players’ hand. If the corresponding card types are known it can also serve as an “open stack”, although TMCG_OpenStack is more suitable in that case.

std::vector<
CardType>
stack [Member of TMCG_Stack]
This is the container that is used internally for storing the cards.

TMCG_Stack () [Constructor on TMCG_Stack]
This default constructor initializes an empty stack.

TMCG_S	ack& = (const TMCG_Stack<
CardType>& that) [Operator on TMCG_Stack]
This is a simple assignment-operator and that is the stack to be assigned.

bool == (const TMCG_Stack<
CardType>& that) [Operator on TMCG_Stack]
This operator tests two stacks for equality. It checks whether the sizes of the stacks and the contained cards are equal with respect to the implied order.

bool != (const TMCG_Stack<
CardType>& that) [Operator on TMCG_Stack]
This operator tests two stacks for inequality. It returns true, if either the sizes do not match or at least two corresponding cards are not equal.

const CardType& [] (const size_t n) [Operator on TMCG_Stack]
This operator provides read-only random access to the contained cards. It returns a const-reference to the nth card from the top of the stack.

CardType& [] (const size_t n) [Operator on TMCG_Stack]
This operator provides random access to the contained cards. It returns a reference to the nth card from the top of the stack.

size_t size () [Method on TMCG_Stack]
This method returns the size of the stack.

void push (const CardType& c) [Method on TMCG_Stack]
This method pushes the card c to the back of the stack.

void push (const TMCG_Stack<
CardType>& s) [Method on TMCG_Stack]
This method pushes the stack s to the back of the stack.
void push (const TMCG_OpenStack<CardType>& s)  
This method pushes the cards of the open stack s to the back of the stack.

bool empty ()  
This method returns true, if the stack is empty.

bool pop (CardType& c)  
This method removes a card from the back and stores the data in c. It returns true, if the stack was not empty and thus c contains useful data.

void clear ()  
This method clears the stack, i.e., it removes all cards.

bool find (const CardType& c)  
This method returns true, if the card c was found in the stack.

bool remove (const CardType& c)  
This method removes the top-most card from the stack which is equal to c. It returns true, if the card was found and successfully removed.

size_t removeAll (const CardType& c)  
This method removes every card from the stack which is equal to c. It returns the number of removed cards.

bool import (std::string s)  
This method imports the stack from the correctly formatted input string s. It returns true, if the import was successful.

~TMCG_Stack ()  
This destructor releases all occupied resources.

std::ostream& << (std::ostream& out, const TMCG_Stack<CardType>& stack)  
This operator exports the given stack to the output stream out.

std::istream& >> (std::istream& in, TMCG_Stack<CardType>& stack)  
This operator imports the given stack from the input stream in. The data has to be delimited by a newline character. The failbit of the stream is set, if any parse error occurred.

TMCG_OpenStack<CardType>  
This struct is a simple container for cards of the specified CardType whose types are known. The elements are pairs where the first component is the type and the second component is the corresponding card. The card type is represented by a size_t integer. Currently, the cards can be either of type TMCG_Card or VTMF_Card depending on which kind of encoding scheme is used.

std::vector<std::pair<size_t, CardType>> stack  
This is the container that is used internally for storing the pairs.

TMCG_OpenStack ()  
This default constructor initializes an empty stack.

TMCG_OpenStack& = (const TMCG_OpenStack<CardType>& that)  
This is a simple assignment-operator and that is the stack to be assigned.
bool == (const TMCG_OpenStack<CardType>& that)
This operator tests two stacks for equality. It checks whether the types, the sizes, and the contained cards are equal with respect to the stack order.

bool != (const TMCG_OpenStack<CardType>& that)
This operator tests two stacks for inequality. It returns true, if either the sizes resp. types do not match or at least two corresponding cards are not equal.

const std::pair<size_t, CardType>& [] (const size_t n)
This operator provides read-only random access to the contained pairs. It returns a const-reference to the nth pair from the top of the stack.

std::pair<size_t, CardType>& [] (const size_t n)
This operator provides random access to the contained pairs. It returns a reference to the nth pair from the top of the stack.

size_t size ()
This method returns the size of the stack.

void push (const std::pair<size_t, CardType>& p)
This method pushes the pair p to the back of the stack. The first component is the type and the second component is the corresponding card representation.

void push (const size_t type, const CardType& c)
This method pushes a pair to the back of the stack. The parameter type is the card type and c is the corresponding card representation.

void push (const TMCG_OpenStack<CardType>& s)
This method pushes the pairs of the stack s to the back of this stack.

bool empty ()
This method returns true, if the stack is empty.

bool pop (size_t& type, CardType& c)
This method removes a pair from the back of the stack. It stores the card type in type and the representation in c. It returns true, if the stack was not empty and thus type and c contain useful data.

void clear ()
This method clears the stack, i.e., it removes all pairs.

bool find (const size_t type)
This method returns true, if a pair with the first component type was found in the stack.

bool remove (const size_t type)
This method removes the top-most pair with the first component type from the stack. It returns true, if such a pair was found and successfully removed.

size_t removeAll (const size_t type)
This method removes every pair from the stack whose first component is equal to type. Further it returns the number of removed pairs.
bool move (const size_t type,  
TMCG_Stack<CardType>& s)  
This method moves the top-most card representation of the given type to another stack s. It returns true, if such a pair was found and successfully moved.

~TMCG_OpenStack ()  
This destructor releases all occupied resources.

TMCG_StackSecret<CardSecretType>  
This struct is a simple container for the secrets involved in the masking operation of cards. Additionally, the permutation of a corresponding shuffle of the stack is stored. The elements are pairs where the first component is a permutation index of type size_t and the second component is a card secret of the specified CardSecretType. Currently, such secrets can be either of type TMCG_CardSecret or VTMF_CardSecret depending on which kind of encoding scheme is used.

std::vector<std::pair<size_t, CardSecretType>> > stack  
This is the container that is used internally for storing the pairs.

TMCG_StackSecret ()  
This default constructor initializes an empty stack secret.

TMCG_StackSecret& = (const  
TMCG_StackSecret<CardSecretType>& that)  
This is a simple assignment-operator and that is the stack secret to be assigned.

const std::pair<size_t, CardSecretType>& [] (const size_t n)  
This operator provides read-only random access to the contained pairs. It returns a constant-reference to the nth pair from the top of the stack secret.

std::pair<size_t, CardSecretType>& [] (const size_t n)  
This operator provides random access to the contained pairs. It returns a reference to the nth pair from the top of the stack secret.

size_t size ()  
This method returns the size of the stack secret.

void push (const size_t index, const  
CardSecretType& cs)  
This method pushes a pair to the back of the stack secret. The parameter index is the permutation index and cs is the corresponding card secret.

void clear ()  
This method clears the stack secret, i.e., it removes all pairs.

size_t find_position (const size_t index)  
This method searches for a given permutation index in the stack secret. It returns the corresponding position5 in the stack secret, if the index was found. Otherwise, the size of the stack secret is returned. Please note that in this case the returned value is not a valid position for an access to the stack secret.

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5 According to the behavior of the []-operator, the zero denotes always the top-most position.
bool find (const size_t index)  
This method searches for a given permutation index in the stack secret. It returns true, if such an index was found.

bool import (std::string s)  
This method imports the stack secret from a correctly formatted input string s. It returns true, if the import was successful.

~TMCG_StackSecret ()  
This destructor releases all occupied resources.

std::ostream& << (std::ostream& out, const TMCG_StackSecret<CardSecretType>& stacksecret)  
This operator exports the given stacksecret to the output stream out.

std::istream& >> (std::istream& in, TMCG_StackSecret<CardSecretType>& stacksecret)  
This operator imports the given stacksecret from the input stream in. The data has to be delimited by a newline character. The failbit of the stream is set, if any parse error occurred.

2.2.1.3 Cryptographic Keys

LibTMCG provides corresponding data types for keys used by the encoding scheme of Schindelhauer [Sc98], because in this scheme it is not efficient to perform the process of key generation for every new game session. These keys are called TMCG keys. However, they also can be utilized to encrypt and sign messages for the more general reasons of confidentiality and integrity, even if the card encoding scheme of Schindelhauer is not used. Therefore these structures may be of independent interest, for example to establish authenticated communication channels between players. However, like for all public key cryptosystems a trusted PKI (Public Key Infrastructure) is needed. This might not be a serious problem in distributed game environments, because the players can include key fingerprints in their individual profile or a service provider can issue public key certificates.

TMCG_SecretKey  
This struct represents the secret part of the key. The underlying public key cryptosystem is due to Rabin (see Digitalized Signatures and Public-Key Functions as Intractable as Factorization, MIT Technical Report 212, 1979) and Williams (see A modification of the RSA public-key encryption procedure, IEEE Transactions on Information Theory, 26(6):726–729, 1980) with minor modifications for encryption padding (SAEP scheme of Boneh [Bo01]) and digital signatures (PRab scheme of Bellare and Rogaway [BR96]).

std::string name  
This string contains the name or a pseudonym of the key owner.

std::string email  
This string contains the email address of the key owner.

std::string type  
This string contains information about the key type. The common prefix is TMCG/RABIN. It is followed by the decimal encoded bit size of the modulus m. The suffix NIZK signals that the correctness of the key is shown by an appended non-interactive zero-knowledge proof. The single parts of the description are separated by underscore characters _, e.g., TMCG/RABIN_2048_NIZK has the correct form. The suffix can be left empty, if the key is only used for encryption and signing (so-called non-NIZK key) without card encoding.
std::string nizk
This string contains two stages of the non-interactive zero-knowledge proof of Gennaro, Micciancio, and Rabin [GMR98]. The proof shows that $m$ was correctly generated as product of at most two primes and both are congruent to 3 (modulo 4). Further there is another non-interactive zero-knowledge proof appended which shows that the condition $y \in \text{NQR}_m^*$ holds.

std::string sig
This string contains a self signature of the public key.

mpz_t m
This is the public modulus $m = p \cdot q$ which is the product of two secret primes $p$ and $q$. The size of $m$ is determined by the security parameter TMCG_QRA_SIZE.

mpz_t y
This is the public quadratic non-residue $y \in \text{NQR}_m^*$ which is used in several zero-knowledge proofs of Schindelhauer’s encoding scheme [Sc98].

mpz_t p
This is the secret prime number $p$ which is a factor of the modulus $m$.

mpz_t q
This is the secret prime number $q$ which is a factor of the modulus $m$.

TMCG_SecretKey ()
This default constructor initializes an empty secret key.

TMCG_SecretKey (const std::string& n, const std::string& e, const unsigned long int keysize = TMCG_QRA_SIZE, const bool nizk_key = true)
This constructor generates a new secret key, where $n$ is the name or a pseudonym of the owner, $e$ is a corresponding email address, $keysize$ is the desired bit length of the modulus $m$, and $nizk_key$ indicates whether or not a NIZK proof will be appended. The default value of the third argument is set to TMCG_QRA_SIZE, if $keysize$ is omitted in the call. The default value of the fourth argument is set to true, whenever it is omitted in the call. Depending on $keysize$ and $nizk_key$ the generation is a very time-consuming task that should be taken into account by the application designer.

TMCG_SecretKey (const std::string& s)
This constructor initializes the key from a correctly formatted input string $s$.

TMCG_SecretKey (const TMCG_SecretKey& that)
This is a simple copy-constructor and $that$ is the key to be copied.

TMCG_SecretKey& = (const TMCG_SecretKey& that)
This is a simple assignment-operator and $that$ is the key to be assigned.

bool check ()
This method tests whether the self signature is valid and whether the non-interactive zero-knowledge proofs are sound. It returns true, if all checks have been successfully passed. Due to the computational complexity of the verification procedure these checks are a very time-consuming task.
std::string fingerprint ()
This method returns the fingerprint of the key. The fingerprint is the hexadecimal notation of the hash value (using algorithm \texttt{TMCG\_GCRY\_MD\_ALGO}) on the concatenated members \texttt{name, email, type, m, y, nizk, and sig}.

std::string selfid ()
This method returns the real value of the self signature. The string \texttt{ERROR} is returned, if any parse error occurred. The string \texttt{SELSIG-SELSIG-SELSIG-SELSIG-SELSIG-SELSIG} is returned, if the self signature \texttt{sig} was empty.

std::string keyid (const size_t size = TMCG\_KEYID\_SIZE)
This method returns the unique key identifier of length \texttt{size}. The default value of the first argument is set to \texttt{TMCG\_KEYID\_SIZE}, if \texttt{size} is omitted in the call.

size_t keyid_size (const std::string& s)
This method returns the length of the unique key identifier \texttt{s}. Zero is returned, if any parse error occurred.

std::string sigid (std::string s)
This method returns the unique key identifier which is included in the signature \texttt{s}. The string \texttt{ERROR} is returned, if any parse error occurred.

bool import (std::string s)
This method imports the key from a correctly formatted input string \texttt{s}. It returns \texttt{true}, if the import was successful.

bool decrypt (unsigned char* value, std::string s)
This method decrypts the given encryption packet \texttt{s} and stores the content in \texttt{value} which is a pointer to a character array of size \texttt{TMCG\_SAEP\_S0}. The method returns \texttt{true}, if the decryption was successful.

std::string sign (const std::string& data)
This method returns a digital signature on \texttt{data}.

std::string encrypt (const unsigned char* value)
This method encrypts the content of \texttt{value} which is a pointer to a character array of size \texttt{TMCG\_SAEP\_S0}. The method returns a corresponding encryption packet that can be decrypted by the owner of the secret key.

bool verify (const std::string& data, std::string s)
This method verifies whether the signature \texttt{s} on \texttt{data} is valid or not. It returns \texttt{true}, if everything was sound.

~TMCG\_SecretKey ()
This destructor releases all occupied resources.

std::ostream& << (std::ostream& out, const TMCG\_SecretKey& key)
This operator exports the given \texttt{key} to the output stream \texttt{out}. 

This operator imports the given key from the input stream `in`. The data has to be delimited by a newline character. The `failbit` is set, if any parse error occurred.

**TMCG_PublicKey**

This `struct` represents the public part of the TMCG key.

- `std::string name` [Member of TMCG_PublicKey]
  - This string contains the name or a pseudonym of the key owner.

- `std::string email` [Member of TMCG_PublicKey]
  - This string contains the email address of the key owner.

- `std::string type` [Member of TMCG_PublicKey]
  - This string contains information about the key type. The common prefix is `TMCG/RABIN`. It is followed by the decimal encoded bit size of the modulus `m`. The suffix `NIZK` signals that the correctness of the key is shown by an appended non-interactive zero-knowledge proof. The single parts of the string are separated by underscore characters `_`, e.g., `TMCG/RABIN_2048_NIZK` has the correct form. However, the suffix can be left empty, if the key is only used for encryption and signing.

- `std::string nizk` [Member of TMCG_PublicKey]
  - This string contains two stages of non-interactive zero-knowledge proof of Gennaro, Micciancio and Rabin [GMR98]. This gives strong evidence that `m` was generated correctly. Further there is another non-interactive zero-knowledge proof appended which shows that the condition `y ∈ NQR_m^2` holds.

- `std::string sig` [Member of TMCG_PublicKey]
  - This string contains the self signature of the public key.

- `mpz_t m` [Member of TMCG_PublicKey]
  - This is the public modulus `m = p \cdot q` which is the product of two secret primes `p` and `q`. The size of `m` is determined by the security parameter `TMCG_QRA_SIZE`.

- `mpz_t y` [Member of TMCG_PublicKey]
  - This is the public quadratic non-residue `y ∈ NQR_m` which is used by several zero-knowledge proofs of the toolbox.

**TMCG_PublicKey ()** [Constructor on TMCG_PublicKey]

This default constructor initializes an empty public key.

**TMCG_PublicKey (const TMCG_SecretKey& skey)** [Constructor on TMCG_PublicKey]

This constructor initializes the key using public values of the secret key `skey`.

**TMCG_PublicKey (const TMCG_PublicKey& pkey)** [Constructor on TMCG_PublicKey]

This is a simple copy-constructor and `pkey` is the key to be copied.

**TMCG_PublicKey& = (const TMCG_PublicKey& that)** [Operator on TMCG_PublicKey]

This is a simple assignment-operator and `that` is the key to be assigned.

**bool check ()** [Method on TMCG_PublicKey]

This method tests whether the self signature is valid and whether the non-interactive zero-knowledge proofs are sound. It returns `true`, if all checks have been successfully passed. Due to the computational complexity of the verification procedure these checks are an extremely time-consuming task.
std::string fingerprint () [Method on TMCG_PublicKey]
This method returns the fingerprint of the key. The fingerprint is the hexadecimal notation of the hash value (using algorithm TMCG_GCRY_MD_ALGO) on the concatenated members name, email, type, m, y, nizk, and sig.

std::string selfid () [Method on TMCG_PublicKey]
This method returns the real value of the self signature. The string ERROR is returned, if any parse error occurred. The string SELFSIG-SELFSIG-SELFSIG-SELFSIG-SELFSIG-SELFSIG is returned, if the self signature sig was empty.

std::string keyid (const size_t size = TMCG_KEYID_SIZE) [Method on TMCG_PublicKey]
This method returns the unique key identifier of length size. The default value of the first argument is set to TMCG_KEYID_SIZE, if size is omitted in the call.

size_t keyid_size (const std::string& s) [Method on TMCG_PublicKey]
This method returns the length of the unique key identifier s. Zero is returned, if any parse error occurred.

std::string sigid (std::string s) [Method on TMCG_PublicKey]
This method returns the unique key identifier which is included in the signature s. The string ERROR is returned, if any parse error occurred.

bool import (std::string s) [Method on TMCG_PublicKey]
This method imports the key from a correctly formatted input string s. It returns true, if the import was successful.

std::string encrypt (const unsigned char* value) [Method on TMCG_PublicKey]
This method encrypts the content of value which is a pointer to a character array of size TMCG_SAEP_S0. The method returns a corresponding encryption packet that can be decrypted by the owner of the secret key.

bool verify (const std::string& data, std::string s) [Method on TMCG_PublicKey]
This method verifies whether the signature s on data is valid or not. It returns true, if everything was sound.

~TMCG_PublicKey () [Destructor on TMCG_PublicKey]
This destructor releases all occupied resources.

std::ostream& << (std::ostream& out, const TMCG_PublicKey& key) [Operator on TMCG_PublicKey]
This operator exports the given key to the output stream out.

std::istream& >> (std::istream& in, TMCG_PublicKey& key) [Operator on TMCG_PublicKey]
This operator imports the given key from the input stream in. The data has to be delimited by a newline character. The failbit is set, if any parse error occurred.

TMCG_PublicKeyRing [Data type]
This struct is just a simple container for TMCG public keys. There are no particular methods provided by TMCG_PublicKeyRing. You have to use the regular interface of the STL container std::vector to access the single keys of the ring.

std::vector<TMCG_PublicKey> keys [Member of TMCG_PublicKeyRing]
This is the real container that is used to store the keys.
TMCG_PublicKeyRing ()
This default constructor initializes an empty public key ring.

TMCG_PublicKeyRing (size_t n)
This constructor initializes the container for storing exactly n keys.

~TMCG_PublicKeyRing ()
This destructor releases all occupied resources.

2.2.2 Communication Interfaces
The base class aiounicast and the corresponding derived classes aiounicast_nonblock and aiounicast_select provide a simple communication interface for asynchronous point-to-point communication channels. They can be used to transfer data of type mpz_t (big integers, see libgmp for explanation of this data type) between up to n parties, that are connected by sockets, pipes or any other file descriptor based input/output mechanism.

Moreover, the channels can be authenticated by a message authentication code and encrypted by a symmetric cipher. The deployed algorithms are defined by global symbols (TMCG_GCRY_MAC_ALGO and TMCG_GCRY_ENC_ALGO, respectively) and fixed at compile time of LibTMCG.

aiounicast
This class is only an abstract interface and cannot be instantiated directly. We explain some basic class members that are useful for an application programmer.

static const time_t aio_timeout_very_short
This constant defines a very short time interval of only one second.

static const time_t aio_timeout_short
This constant defines a short time interval of 15 seconds.

static const time_t aio_timeout_middle
This constant defines a middle time interval of 30 seconds.

static const time_t aio_timeout_long
This constant defines a long timeout interval of 90 seconds.

static const time_t aio_timeout_very_long
This constant defines a very long time interval of 180 seconds.

static const time_t aio_timeout_extremly_long
This constant defines an extremely long time interval of 300 seconds.

static const size_t aio_scheduler_roundrobin
This constant represents the round-robin scheduler for message processing.

static const size_t aio_scheduler_random
This constant represents the random select scheduler for message processing.

static const size_t aio_scheduler_direct
This constant represents the constant select scheduler for message processing.

const size_t n
This is the total number of parties n involved in the communication.

const size_t j
This is an unique index of the party running this instance.
std::map<size_t, int> fd_in
The input file descriptors of point-to-point links to all parties.

std::map<size_t, int> fd_out
The output file descriptors of point-to-point links to all parties.

size_t numWrite
The total number of bytes written to point-to-point links.

size_t numRead
The total number of bytes read from point-to-point links.

size_t numEncrypted
The total number of bytes that have been encrypted yet.

size_t numDecrypted
The total number of bytes that have been decrypted yet.

size_t numAuthenticated
The total number of bytes that have been authenticated yet.

Note that the header files aiounicast_nonblock.hh or aiounicast_select.hh must be included in addition to libTMCG.hh. The use of class aiounicast_select is strongly recommended.

aiounicast_nonblock
This class works with non-blocking file descriptors, i.e., the pipes or sockets have to be opened with the O_NONBLOCK flag. The methods use continuous polling on the descriptors to achieve asynchronous I/O that results in exorbitant CPU load. The class should be used only, if no select system call is available or appropriate for the application.

aiounicast_nonblock (const size_t n_in, const size_t j_in, const std::vector<int>& fd_in, const std::vector<int>& fd_out, const std::vector<std::string>& key_in, const size_t aio_default_scheduler_in = aio_scheduler_roundrobin, const time_t aio_default_timeout_in = aio_timeout_long, const bool aio_is_authenticated_in = true, const bool aio_is_encrypted_in = true)
The constructor initializes internal queues and data structures for asynchronous point-to-point channels connecting n parties (i.e. n_in). The index of the calling party within this set is given by j_in. It is followed by a vector fd_in of exactly n input file descriptors that are ready for reading and writing, and by a vector fd_out of exactly n output file descriptors. Finally, the vector key_in with exactly n passphrases or pre-shared keys is necessary, if aio_is_authenticated_in or aio_is_encrypted_in is set true, which is the default behaviour. The default values for timeout (in seconds) and the receive scheduler can be modified carefully according to the desired usage scenario.

bool Send (mpz_srcptr m, const size_t i_in, time_t timeout = aio_timeout_default)
This method sends an integer m over the corresponding point-to-point link to the party with index i_in. In presence of the third argument this transmission is tried for at most timeout seconds. Otherwise, the default timeout given to the constructor is applied.

The method returns false, if sending fails, and error messages are written to std::cerr.

The key derivation function PBKDF2 is applied with an iteration count of 25,000 and a different constant salt to derive the authentication and the encryption key, respectively.
bool Send (const std::vector<mpz_srcptr>& m, const size_t i_in, time_t timeout =aio_timeout_default)
This method works as above, however, a vector m of integers is sent.

bool Receive (mpz_ptr m, size_t& i_out, size_t scheduler =aio_scheduler_default, time_t timeout =aio_timeout_default)
This method receives an integer m over the point-to-point links from any party. The index of the sender is returned in i_out. In presence of the third argument it waits for at most timeout seconds. Otherwise, the default timeout given to the constructor is applied.

The method returns false, if receiving fails. Only in critical cases some error messages are written to std::cerr.

bool Receive (std::vector<mpz_ptr>& m, size_t& i_out, size_t scheduler =aio_scheduler_default, time_t timeout =aio_timeout_default)
This method works as above, however, a vector m of integers is received.

~aiounicast_nonblock ()
This destructor releases all occupied resources.

aiounicast_select
This class works with arbitrary file descriptors. It uses the select interface of the operation system with negligible timeout of 1000us to achieve asynchronous I/O. This results in a reasonable CPU load in comparison with aiounicast_nonblock.

aiounicast_select (const size_t n_in, const size_t j_in, const std::vector<int>& fd_in_in, const std::vector<int>& fd_out_in, const std::vector<std::string>& key_in, const size_t aio_default_scheduler_in =aio_scheduler_roundrobin, const time_t aio_default_timeout_in =aio_timeout_long, const bool aio_is_authenticated_in =true, const bool aio_is_encrypted_in =true)
The constructor initializes internal queues and data structures for asynchronous point-to-point channels connecting n parties (i.e. n_in). The index of the calling party within this set is given by j_in. It is followed by a vector fd_in_in of exactly n input file descriptors that are ready for reading and writing, and by a vector fd_out_in of exactly n output file descriptors. Finally, the vector key_in with exactly n passphrases7 or pre-shared keys is neccessary, if aio_is_authenticated_in or aio_is_encrypted_in is set true, which is the default behaviour. The default values for timeout (in seconds) and the receive scheduler can be modified carefully according to the desired usage scenario.

bool Send (mpz_srcptr m, const size_t i_in, time_t timeout =aio_timeout_default)
This method sends an integer m over the corresponding point-to-point link to the party with index i_in. In presence of the third argument this transmission is tried for at most timeout seconds. Otherwise, the default timeout given to the constructor is applied.

The method returns false, if sending fails, and error messages are written to std::cerr.

7 The key derivation function PBKDF2 is applied with an iteration count of 25,000 and a different constant salt to derive the authentication and the encryption key, respectively.
bool Send (const std::vector<mpz_srcptr>& m, const size_t i_in, time_t timeout = aio_timeout_default)
This method works as above, however, a vector m of integers is sent.

bool Receive (mpz_ptr m, size_t& i_out, size_t scheduler = aio_scheduler_default, time_t timeout = aio_timeout_default)
This method receives an integer m over the point-to-point links from any party. The index of the sender is returned in i_out. In presence of the third argument it waits for at most timeout seconds. Otherwise, the default timeout given to the constructor is applied.
The method returns false, if receiving fails. Only in critical cases some error messages are written to std::cerr.

bool Receive (std::vector<mpz_ptr>& m, size_t& i_out, size_t scheduler = aio_scheduler_default, time_t timeout = aio_timeout_default)
This method works as above, however, a vector m of integers is received.

~aiounicast_select ()
This destructor releases all occupied resources.

2.2.3 Classes
LibTMCG consists of several C++ classes. Some of them are only extensions or optimizations, but other provide necessary interfaces to perform the basic operations in secure card games, e.g., the creation of open cards, the masking of cards, the opening of masked cards, the verifiable secret shuffle of a stack, and more general tasks like distributed key generation procedures. Each class implements the some functionality of the corresponding research paper [CKPS01, BS03, JL00, Gr05, HSSV09, Sc98]. The author names are a prefix of the class name and the following part is an abbreviation of (a part of) the title, respectively.

2.2.3.1 Secure and Efficient Asynchronous Broadcast Protocols
This part of LibTMCG provides an implementation of reliable broadcast, which is actually based on an optimized variant of Bracha’s double-echo broadcast protocol. It works without further authentication mechanisms (e.g. digital signatures) and thus guarantees the desired properties (i.e. validity, consistency, and totality\(^8\)) of reliable broadcast only, if the number of faulty or even malicious players t is strictly less than one third of all parties n, i.e. \(t < n/3\). Please note that without further assumptions this condition is rather optimal for asynchronous communication and thus has crucial impact for liveness of the high-level protocols using it.

We describe only those classes, methods, and members that might be of interest for an application programmer.

CachinKursawePetzoldShoupRBC [Class]
This class implements the protocol RBC by Cachin, Kursawe, Petzold, and Shoup [CKPS01] for a reliable broadcast in the asynchronous communication model, where \(t < n/3\) holds. Additionally, a FIFO-ordered delivery mechanism based on sequence numbers has been implemented.

size_t n [Member of CachinKursawePetzoldShoupRBC]
This is the total number of parties n involved in this protocol.

\(^8\) Totality ensures that all correct parties either deliver a message or don’t. In the literature consistency and totality properties are often combined into a single condition called agreement.
size_t t  
This is the number of possible faulty parties $t$.

size_t j  
This is an unique index of the party running this instance.

CachinKursawePetzoldShoupRBC  
[Constructor on CachinKursawePetzoldShoupRBC]
(const size_t n_in, const size_t t_in, const size_t j_in, aiounicast*
aiou_in, const size_t aio_default_scheduler_in
=aiounicast::aio_scheduler_roundrobin, const time_t
aio_default_timeout_in =aiounicast::aio_timeout_very_long)
The constructor initializes an instance for a reliable broadcast channel of $n$ parties. This
total number of parties is given in the first argument $n_{in}$. The number of possible faulty
or even malicious parties $t$ (given in the second argument $t_{in}$) must not exceed
$n/3$. Otherwise a warning is printed to std::cerr and the liveness of the protocol RBC is not
guaranteed. Thus, it is recommended to set $t_{in}$ to the asynchronous maximum $(n-1)/3$.
The third argument $j_{in}$ is an index of the party running this instance. Finally, the
constructor needs as fourth argument $aiou_{in}$ a reference to already established point-
to-point channels (see Section 2.2.2 [Communication Interfaces], page 24), which should
exclusively\(^9\) used for this broadcast channels. The default values for timeout (in seconds)
and the deliver scheduler can be modified carefully with respect to the usage scenario.

void setID (const std::string ID_in)  
[Method on CachinKursawePetzoldShoupRBC]
Broadcast channels are parameterized by a tag called ID, that is contained in every mes-
message. This method sets the tag to $ID_{in}$, which should be equal for all parties for the
desired channel.

void unsetID ()  
[Method on CachinKursawePetzoldShoupRBC]
This method unset the current channel tag and returns to the previous value. This is
commonly used to return from a channel of a subprotocol to the channel of the calling
protocol.

void Broadcast (mpz_srcptr m,  
const bool simulate_faulty behaviour =false)  
[Method on CachinKursawePetzoldShoupRBC]
This method broadcasts the integer $m$ to all parties.

bool Deliver (mpz_ptr m, size_t& i_out,  
scheduler =aiounicast::aio_scheduler_default, time_t
timeout =aiounicast::aio_timeout_default)  
[Method on CachinKursawePetzoldShoupRBC]
This method delivers a broadcasted integer $m$ from any party using deliver scheduler
scheduler. The index of the sender is returned in $i_{out}$. In presence of the fourth argu-
ment it waits for at most $timeout$ seconds. Otherwise, the default timeout given to the
constructor is applied.
The method returns false, if delivering fails. Only in some critical cases error messages
are written to std::cerr.

bool DeliverFrom (mpz_ptr m,  
const size_t i_in, scheduler =aiounicast::aio_scheduler_default, time_t
timeout =aiounicast::aio_timeout_default)  
[Method on CachinKursawePetzoldShoupRBC]
This method delivers a broadcasted integer $m$ from a specified party with index $i_{in}$ using
deliver scheduler scheduler. In presence of the fourth argument it waits at most for $timeout$
seconds. Otherwise, the default timeout given to the constructor is applied.

\(^9\) These channels should be authenticated such that network attacks or errors can be detetcted.
The method returns `false`, if delivering fails. Only in some critical cases error messages are written to `std::cerr`.

```cpp
bool Sync (time_t timeout = aiounicast::aio_timeout_default, const std::string tag = "")
```

This method continues the execution of RBC protocol such that the requests of other waiting parties are satisfied. In presence of the first argument it waits approximately for \((t+1) \cdot \text{timeout}\) seconds while trying to synchronize all parties based on their corresponding local Unix Epoch time. Otherwise, the default timeout given to the constructor is applied. Each synchronization point is required to be unique. Thus, a string called `tag` with a description of the synchronization point can be supplied as second argument of this method.

The method returns `false`, if synchronization is failed.

```cpp
~CachinKursawePetzoldShoupRBC ()
```

This destructor releases all occupied resources.

### 2.2.3.2 Verifiable k-out-of-k Threshold Masking Function

The two classes of this subsection are concrete instantiations of Barnett and Smart’s VTMF primitive [BS03]. More formally, the authors specify four different protocols:

- Key Generation Protocol
- Verifiable Masking Protocol
- Verifiable Re-masking Protocol
- Verifiable Decryption Protocol

Each protocol uses low-level operations on an appropriately chosen algebraic group \(G\). The choice of this group is crucial to the security of the card encoding scheme and thus also to the security of high-level operations on cards resp. stacks.

There are just a few methods and members of these classes that might be of general interest for an application programmer, e.g. the methods of the key generation protocol. The other stuff is only used internally by high-level operations of SchindelhauerTMCG. Therefore this manual omits the description of such internal functions and members.

**BarnettSmartVTMF_dlog**

This class implements the discrete logarithm instantiation of the VTMF primitive in the field \(\mathbb{Z}/p\mathbb{Z}\), where \(p\) is a large prime number. The mathematical computations are performed in the finite cyclic subgroup \(G\) of prime order \(q\) such that \(p = kq + 1\) holds for some \(k \in \mathbb{Z}\). The security relies on the DDH assumption in \(G\), i.e., the distribution \(\{g^a, g^b, g^{ab}\}\) is computationally indistinguishable from \(\{g^a, g^b, g^c\}\), where \(g\) is a generator of \(G\) and \(a, b, c\) are chosen at random from \(\mathbb{Z}_q\). Currently, this well-established assumption is believed to hold, if \(p\) and \(q\) are chosen according to the predefined security parameters of LibTMCG.

```cpp
mpz_t p
```

This is the public prime number \(p\) which defines the underlying finite field \(\mathbb{Z}/p\mathbb{Z}\).

```cpp
mpz_t q
```

This is the public prime number \(q\) which defines the underlying cyclic group \(G\). \(G\) is a subgroup of \(\mathbb{Z}/p\mathbb{Z}\) and is exactly of order \(q\).

```cpp
mpz_t g
```

This is the fixed public generator \(g\) of the underlying group \(G\).
This is a public integer $k$ such that $p = kq + 1$ holds.

This is the common public key $h = \prod_{i=1}^{k} h_i$ which contains the public keys $h_i$ of each player $P_i$. Note that in the above formula $k$ denotes the number of players.

This is the public key $h_i$ of this player instance.

This constructor creates a new VTMF instance. That means, the primes $p$ and $q$ are randomly and uniformly chosen such that they have length $\text{fieldsize}$ bit and $\text{subgroupsize}$ bit, respectively. Further, either a generator $g$ for the unique subgroup of order $q$ is chosen at random or, if $\text{canonical} \_g \_usage$ is set true, the generator $g$ is chosen in a verifiable way (cf. FIPS 186-3 A.2.3). If the arguments are omitted, then $\text{fieldsize}$, $\text{subgroupsize}$ and $\text{canonical} \_g \_usage$ are set to their default values $\text{TMCG} \_\text{DDH} \_\text{SIZE}$, $\text{TMCG} \_\text{DLSE} \_\text{SIZE}$, and false, respectively. The argument $\text{initialize} \_\text{group}$ should be always set true. Depending on $\text{fieldsize}$ and $\text{subgroupsize}$ the group generation is a very time-consuming task that should be taken into account by the application designer.

This constructor initializes the VTMF instance from a correctly formatted input stream $\text{in}$. For example, such a stream can be generated by calling the method $\text{PublishGroup}$ of an already created instance. The arguments $\text{fieldsize}$, $\text{subgroupsize}$, and $\text{canonical} \_g \_usage$ are set to their default values $\text{TMCG} \_\text{DDH} \_\text{SIZE}$, $\text{TMCG} \_\text{DLSE} \_\text{SIZE}$, and $\text{false}$, respectively. The argument $\text{precompute}$ should be always set true. If these arguments are omitted, then they are set to the default values $\text{TMCG} \_\text{DDH} \_\text{SIZE}$, $\text{TMCG} \_\text{DLSE} \_\text{SIZE}$, $\text{false}$, and true respectively.

This method checks whether $p$ and $q$ have appropriate sizes with respect to the bit lengths given during the initialization of the corresponding instance. Further, it checks whether $p$ has the correct form (i.e. $p = kq + 1$), whether $p$ and $q$ are probable prime, and whether $g$ is a generator of the subgroup $G$. If $\text{canonical} \_g \_usage$ is set true during the call of constructor, then it additionally checks whether $g$ was generated in a verifiable way (cf. FIPS 186-3 A.2.3). It returns true, if all of these checks have been passed successfully.

This method exports all necessary group parameters of $G$ to the given output stream $\text{out}$. For example, such a stream can be generated by calling the method $\text{PublishGroup}$ of an already created instance. The arguments $\text{fieldsize}$, $\text{subgroupsize}$, and $\text{canonical} \_g \_usage$ are stored for later following usage, e.g. by the method $\text{CheckGroup}$ as explained below. The argument $\text{precompute}$ should be always set true. If these arguments are omitted, then they are set to the default values $\text{TMCG} \_\text{DDH} \_\text{SIZE}$, $\text{TMCG} \_\text{DLSE} \_\text{SIZE}$, $\text{false}$, and true respectively.

This method generates a VTMF key pair and stores the numbers internally for a later following usage. It must be called before any other part of the key generation protocol is executed. Otherwise, the produced results are wrong.
void KeyGenerationProtocol_PublishKey (std::ostream& out)

This method exports the public part $h_i$ of the generated VTMF key pair to the given output stream out. Further, it appends a non-interactive zero-knowledge proof of knowledge (NIZK) which shows that the instance knows the secret part $x_i$ such that $h_i \equiv g^{x_i} \pmod{p}$ holds. Due to the non-interactive nature of this proof the method has to be called only once while the computed output can be reused multiple times if necessary.

bool KeyGenerationProtocol_UpdateKey (std::istream& in)

This method reads the public part of a VTMF key and the NIZK from the input stream in. It appends the key to the common public key and returns true, if the given proof was sound. Otherwise, false is returned.

bool KeyGenerationProtocol_RemoveKey (std::istream& in)

This method reads the public part of a VTMF key and the corresponding NIZK from the input stream in. It removes the key from the common public key and returns true, if the key was previously appended by KeyGenerationProtocol_UpdateKey as explained above.

void KeyGenerationProtocol_Finalize ()

This method must be called after any update (KeyGenerationProtocol_UpdateKey) or removal (KeyGenerationProtocol_RemoveKey) has been performed on the common public key.

~BarnettSmartVTMF_dlog ()

This destructor releases all occupied resources.

BarnettSmartVTMF_dlog_GroupQR

This subclass implements the discrete logarithm instantiation of the VTMF primitive in the field $\mathbb{Z}/p\mathbb{Z}$, where $p$ is a large prime number. The mathematical computations are performed in a special finite cyclic subgroup $G$ (quadratic residues modulo $p$) of prime order $q$, where $p = 2q + 1$ holds. The security also relies on the DDH assumption w.r.t. $G$, i.e., the distribution $\{g^a, g^b, g^{ab}\}$ is computationally indistinguishable from $\{g^a, g^b, g^c\}$, where $g$ is a generator of $G$ and $a, b, c$ are chosen at random from $\mathbb{Z}_q$. Currently, this well-established assumption is believed to hold, if $p$ and $q$ are chosen according to the predefined security parameters of LibTMCG.

mpz_t p

This is the public prime number $p$ which defines the underlying finite field $\mathbb{Z}/p\mathbb{Z}$.

mpz_t q

This is the public prime number $q$ which defines the underlying cyclic group $G$. $G$ denotes the unique subgroup of quadratic residues modulo $p$ which is exactly of order $q$, if $p = 2q + 1$ holds.

mpz_t g

This is the fixed public generator $g$ of the underlying group $G$.

mpz_t k

This integer is fixed here by $k = 2$.

mpz_t h

This is the common public key $h = \prod_{i=1}^k h_i$ which contains the public keys $h_i$ of each player $P_i$. Note that in the above formula $k$ denotes the number of players.
This is the public key $h_i$ of this player instance.

This constructor creates a new VTMF instance. That means, the safe prime $p$ is randomly and uniformly chosen such that it has a length of $\text{fieldsize}$ bit. Further, the generator $g$ is initially set up by 2 and then shifted by $\text{fieldsize} - \text{exponentsize}$ bit positions, according to the procedure described by Koshiba and Kurosawa (see Short Exponent Diffie-Hellman Problems, PKC 2004, LNCS 2947). If the arguments of the constructor are omitted, then $\text{fieldsize}$ and $\text{exponentsize}$ are set to their default values $\text{TMCG\_DDH\_SIZE}$ and $\text{TMCG\_DLSE\_SIZE}$, respectively. Depending on $\text{fieldsize}$ and $\text{exponentsize}$ the group generation is a very time-consuming task that should be taken into account by the application designer.

This constructor initializes the VTMF instance from a correctly formatted input stream $\text{in}$. For example, such a stream can be generated by calling the method $\text{PublishGroup}$ of an already created instance. The arguments $\text{fieldsize}$ and $\text{exponentsize}$ are stored for later following usage, e.g. by the method $\text{CheckGroup}$ as explained below. If these arguments are omitted, then they are set to the default values $\text{TMCG\_DDH\_SIZE}$ and $\text{TMCG\_DLSE\_SIZE}$, respectively.

This method checks whether $p$ and $q$ have appropriate sizes with respect to the bit lengths given during the initialization of the corresponding instance. Further, it checks whether $p$ has the correct form (i.e. $p = 2q + 1$), whether $p$ and $q$ are probable prime, and whether $g$ is a generator of the subgroup $G$. It returns $true$, if all of these checks have been passed successfully.

This destructor releases all occupied resources.

2.2.3.3 Adaptively Secure Threshold Cryptography

Jarecki and Lysyanskaya [JL00] have introduced some useful building blocks in order to gain security against an adaptive adversary for threshold cryptography.

This class provides the erasure-free distributed coinflip (EDCF) protocol. It also needs a group $G_q$ of prime order $q$ where the discrete logarithm problem is computationally hard. The protocol produces a public value $a = \sum_{i=1}^{n} a_i \mod q$ such that $0 \leq a < q$ is random and uniformly distributed, if at least one party $P_i, 1 \leq i \leq n$ has chosen their corresponding coin share $a_i \in \mathbb{Z}_q$ uniformly at random.

The coinflip protocol is useful in order to transform a public-coin honest-verifier zero-knowledge proof of knowledge (HVZKP) into interactive proof resp. argument which preserve the zero-knowledge property even in case of malicious verifiers. Such proof systems are called simultaneous zero-knowledge proofs of knowledge. The underlying general model of Jarecki and Lysyanskaya [JL00] considers a synchronous communication network of $n$ players with access to a reliable broadcast channel, where an adaptive adversary can corrupt up to a minority $t < n/2$ of the players.
This is the public prime number $p$ which defines the underlying finite field $\mathbb{Z}/p\mathbb{Z}$.

This is the public prime number $q$ which defines the underlying cyclic group $G_q$. Note that $G_q$ is a subgroup of $\mathbb{Z}/p\mathbb{Z}$ and it must be chosen to have order $q$.

This is the fixed public generator $g$ of the underlying group $G_q$.

This is the common public value $h \in G_q$ such that nobody knows $\log_g h$. It can be obtained by the above key generation protocol (see Section 2.2.3.2 [BarnettSmartVTMF], page 29).

Jarecki and Lysyanskaya [JL00]: “When secure channels are present, $h$ can be obtained by using general techniques of multi-party computation [BGW88, CDD+99]. When secure channel are not there, and implementing them by erasure is not an option, we can use another protocol, where each player generates his share $h_i$ of $h$, and then all players, in parallel, prove knowledge of $\log_g h_i$ to each other.”

This is the total number of parties $n$ involved in this protocol.

This is the maximum number of faulty parties $t$ (reconstruction threshold).

This constructor creates a new EDCF instance. That means, the required primes $p$ and $q$ and the generators $g$ and $h$ are initialized from the given arguments $p_{CRS}$, $q_{CRS}$, $g_{CRS}$, and $h_{CRS}$, respectively. $n_{in}$ is the total number of participating players, for which at most $t_{in}$ are faulty or act malicious during the protocol execution.

This method checks whether $p$ and $q$ have appropriate sizes with respect to the bit lengths given during the initialization of the corresponding instance. Further, it checks whether $p$ has the correct form (i.e. $p = kq + 1$), whether $p$ and $q$ are probable prime, and whether $g$ resp. $h$ are different generators of the subgroup $G_q$. It returns true, if all of these checks have been passed successfully.

This method starts the protocol which produces a public value $a = \sum_{i=1}^{n} a_i \mod q$ such that $0 \leq a < q$ is random and uniformly distributed, if at least one party $P_i$, $1 \leq i \leq n$ has chosen their corresponding share $a_i \in \mathbb{Z}_q$ uniformly at random. If it returns true, then $a$ contains this common random value. The argument $i$ is an index of the running instance with respect to already initialized instances of asynchronous point-to-point channels $aiou$ and a reliable broadcast channel $rbc$. Logging and debug messages are printed to the provided output stream $err$. 
bool Flip_twoparty (const size_t i, mpz_ptr a, std::istream& in, std::ostream& out, std::ostream& err, const bool simulate_faulty Behaviour = false)

This is the two-party version of the above method. Thus there are only an input stream in and output stream out for communication between the players. The other arguments are as above.

~JareckiLysyanskayaEDCF ()

This destructor releases all occupied resources.

2.2.3.4 Verifiable Secret Shuffle of Homomorphic Encryptions

Recently, Groth [Gr05, Gr10] has proposed a very efficient solution to perform a verifiable shuffle of homomorphically encrypted values. He describes an honest verifier zero-knowledge argument which shows the correctness of a shuffle. Beside other applications (e.g. verifiable mix networks, electronic voting) his protocol can be used to show (with overwhelming probability) that the secret shuffle of a deck of cards was performed correctly. The computational complexity and the produced communication traffic are superior to previously deployed techniques (e.g. Schindelhauer’s cut-and-choose method). LibTMCG provides the first known implementation of Groth’s famous protocol. However, it can only be used along with the VTMF card encoding scheme of Barnett and Smart [BS03] based on the hardness of computing discrete logarithms.

Our implementation uses a generalized variant [Gr05, Gr10] of the statistically hiding and computationally binding homomorphic commitment scheme due to Pedersen (see Non-interactive and Information-theoretic Secure Verifiable Secret Sharing, CRYPTO ’91, LNCS 576, 1992). The binding property relies on the hardness of computing discrete logarithms in $G$ w.r.t. random bases $g_1, \ldots, g_n$ and thus a commitment is only binding for computationally bounded provers. But this choice seems to be reasonable for the intention of LibTMCG, because all players are supposed to be computationally bounded. The security parameters of the commitment scheme (in particular the group $G$) are determined by the corresponding VTMF instance.

Since version 1.2.0 of LibTMCG we use a two-party version of a distributed coin flipping protocol by Jarecki and Lysyanskaya [JL00] to protect against malicious verifiers attacking the zero-knowledge property. Since version 1.3.0 there is an additional method for generating the bases $g_1, \ldots, g_n$ of the Pedersen commitment scheme by distributed coin flipping and a verifiable generation procedure similar to FIPS 186-3 A.2.3. This step is important in order to ensure, that a malicious prover cannot compute $\log_{g_i} h$ resp. $\log_h g_i$, for some $i = 1, \ldots, n$, and thus erroneously pass the shuffle verification. It improves our former security model which considered only a passive adversary.

Further, to the best of our knowledge it is not known, whether Groth’s protocol retains the zero-knowledge property when it is executed in a concurrent setting. Thus the application programmer should be careful and avoid parallel invocations of the same instance.

GrothVSSHE

This class provides the low-level interface for Groth’s protocol. There are just a few methods that might be of general interest. All other components are only used internally by high-level operations and thus their description is omitted here.

10 Strictly speaking, due to this reason Groth’s protocol is a zero-knowledge argument instead of a zero-knowledge proof. However, for convenience we will not distinguish between these terms here.
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**GrothVSSHE** (size_t n, mpz_srcptr p_ENC, [Constructor on GrothVSSHE]
mpz_srcptr q_ENC, mpz_srcptr k_ENC, mpz_srcptr g_ENC,
mpz_srcptr h_ENC, unsigned long int ell_e = TMCG_GROTH_L_E,
unsigned long int fieldsize = TMCG_DDH_SIZE, unsigned long int
subgroupsise = TMCG_DLSE_SIZE)

This constructor creates a new instance. The low-level operations are later used to show
the correctness of a shuffle of at most \( n \) cards. The protocol and some parameters of the
commitment scheme are initialized by the members of the corresponding VTMF instance.
Consequently, \( p_{\text{ENC}} \) is the prime number \( p \) which determines the field \( \mathbb{Z}/p\mathbb{Z} \), \( q_{\text{ENC}} \)
is the order of the underlying subgroup \( G \), i.e. the prime number \( q \), and \( k_{\text{ENC}} \) is the
integer such that \( p = qk + 1 \) holds. Further, \( g_{\text{ENC}} \) is the generator \( g \) of this subgroup,
and finally \( h_{\text{ENC}} \) is the common public key \( h \). The positive integer \( \ell_{\text{ell}} \) is the security
parameter which controls the soundness error probability \( (2^{-\ell_{\text{ell}}}) \) of the protocol. The
default value is defined by \( \text{TMCG}_\text{GROTH}_L_E \), if this argument is omitted. The \( \text{fieldsise} \)
and the \( \text{subgroupsise} \) are supplied to internal classes and are only of interest, if \( p_{\text{ENC}} \)
or \( q_{\text{ENC}} \) have lengths different from the default. If these arguments are omitted, they
are set to \( \text{TMCG}_\text{DHH}_\text{SIZE} \) and \( \text{TMCG}_\text{DLSE}_\text{SIZE} \), respectively.

This constructor should be instantiated only once by the session leader. All other instances
can be created by the second constructor. Further, it is very important that the VTMF key
generation protocol has been finished before the value of \( h \) is passed to the constructors.
Otherwise, the correctness verification of the shuffle will fail.

Note that the generators \( g_1, \ldots, g_n \) of the Pedersen commitment scheme are randomly and
uniformly chosen from \( \mathbb{Z}_q \) by the session leader. However, this is not verifiable by other
parties and a malicious leader can choose \( g_j := h^{\ell_{\text{ell}}} \mod p \) for some secret \( \xi_j \in \mathbb{Z}_q \)
where \( 1 \leq j \leq n \). Thus it is important to call **SetupGenerators_publiccoin** during game
initialization before any shuffle verification is performed.

**GrothVSSHE** (size_t n, std::istream& in, unsigned [Constructor on GrothVSSHE]
long int ell_e = TMCG_GROTH_L_E, unsigned long int fieldsise
= TMCG_DHH_SIZE, unsigned long int subgroupsise = TMCG_DLSE_SIZE)

This constructor initializes the instance from a correctly formatted input stream \( in \). For
example, such a stream can be generated by calling the method **PublishGroup** of an
already created instance. Later the instance can be used to show the correctness of a shuffle
of at most \( n \) cards. The positive integer \( \ell_{\text{ell}} \) controls the soundness error probability of the
protocol. The default value is defined by \( \text{TMCG}_\text{GROTH}_L_E \), if this argument is omitted.

Note that the generators \( g_1, \ldots, g_n \) of the Pedersen commitment scheme are randomly and
uniformly chosen from \( \mathbb{Z}_q \) by the session leader. However, this is not verifiable by other
parties and a malicious leader can choose \( g_j := h^{\ell_{\text{ell}}} \mod p \) for some secret \( \xi_j \in \mathbb{Z}_q \)
and \( 1 \leq j \leq n \). Thus it is necessary to call the method **SetupGenerators_publiccoin** before
any shuffle verification is performed.

void SetupGenerators_publiccoin (mpz_srcptr a) [Method on GrothVSSHE]

This is a simple method to setup the generators \( g_1, \ldots, g_n \) of the internal Pedersen commit-
tment scheme by using a common random value \( a \) for a verifiable generation procedure similar
to FIPS 186-3 A.2.3. Note that the same \( a \) must be used by all participants and that
this value should be different for each game session.

bool SetupGenerators_publiccoin (size_t whoami, [Method on GrothVSSHE]
aiounicast* aiou, CachinKursavePetzoldShoupRBC* rbc,
JareckiLysyanskayaEDCF* edcf, std::ostream& err)

This method setup the generators \( g_1, \ldots, g_n \) of the internal Pedersen commitment scheme
by using a distributed coinflip protocol [JL00] and a verifiable generation procedure similar
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Table 2.1: VTMF Key Generation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>Prime number determining the field $\mathbb{Z}/p\mathbb{Z}$</td>
</tr>
<tr>
<td>$q$</td>
<td>Order of the underlying subgroup $G$</td>
</tr>
<tr>
<td>$k$</td>
<td>Integer such that $p = qk + 1$ holds</td>
</tr>
<tr>
<td>$g$</td>
<td>Generator $g$ of the subgroup $G$</td>
</tr>
<tr>
<td>$h$</td>
<td>Common public key $h$</td>
</tr>
<tr>
<td>$f\text{ldsize}$</td>
<td>Field size</td>
</tr>
<tr>
<td>$s\text{ubgroupsize}$</td>
<td>Subgroup size</td>
</tr>
</tbody>
</table>

2.2.3.5 Verifiable Rotation of Homomorphic Encryptions

De Hoogh, Schoenmakers, Skoric, and Villegas [HSSV09] have proposed an efficient solution to perform a verifiable rotation (also known as cyclic shift) of homomorphically encrypted values. Other solutions (e.g., Reiter and Wang, Fragile Mixing, ACM CCS, 2004) do not provide that level of efficiency. LibTMCG provides the first known implementation of their protocol. It can only be used with the VTMF card encoding scheme of Barnett and Smart [BS03].

Further, to the best of our knowledge it is not known, whether their protocol retains the zero-knowledge property when it is executed in a concurrent setting. Thus the application programmer should be careful and avoid parallel invocations of the same instance.

HooghSchoenmakersSkoricVillegasVRHE

This class provides the low-level interface for their protocol. There are just a few methods that might be of general interest. All other components are only used internally by high-level operations and thus their description is omitted here.

HooghSchoenmakersSkoricVillegasVRHE on HooghSchoenmakersSkoricVillegasVRHE

This constructor creates a new instance. The low-level operations are later used to show the correctness of a rotation of the cards. The protocol and some of its parameters are initialized by the members of the corresponding VTMF instance. Consequently, $p\_ENC$ is the prime number $p$ which determines the field $\mathbb{Z}/p\mathbb{Z}$, $q\_ENC$ is the order of the underlying subgroup $G$, i.e., the prime number $q$, and $k\_ENC$ is the integer such that $p = qk + 1$ holds. Further, $g\_ENC$ is the generator $g$, and finally $h\_ENC$ is the common public key $h$. The $f\text{ldsize}$ and the $s\text{ubgroupsize}$ are supplied to internal classes and are only of interest, if $p\_ENC$ or $q\_ENC$ have lengths different from the default. If these arguments are omitted, they are set to $T\text{MC}\_D\text{DH}\_S\text{IZE}$ and $T\text{MC}\_D\text{LSE}\_S\text{IZE}$, respectively.

This constructor should be instantiated only once by the session leader. All other instances must be created by the second constructor. Further, it is very important that the VTMF key generation protocol has been finished before the value of $h$ is passed to the constructor. Otherwise, the correctness verification will definitely fail.
This constructor initializes the instance from a correctly formatted input stream `in`. For example, such a stream can be generated by calling the method `PublishGroup` of an already created instance. Later the instance can be used to show the correctness of a rotation.

```cpp
bool CheckGroup ()
```
This method checks whether the initialized commitment scheme is sound. It returns `true`, if all tests have been passed successfully.

```cpp
void PublishGroup (std::ostream& out)
```
This method exports the instance configuration to the output stream `out` such that other instances can be initialized, e.g., with the second constructor.

```cpp
~HooghSchoenmakersSkoricVillegasVRHE()
```
This destructor releases all occupied resources.

### 2.2.3.6 Toolbox for Mental Card Games

This section explains the main class of LibTMCG which provides some “high-level operations” from Schindelhauer’s toolbox [Sc98]. Even if the more efficient card encoding scheme of Barnett and Smart [BS03] will deployed in your application, at least one instance of the following class must be created to perform any card or stack operations.

**SchindelhauerTMCG**

This class implements the main core of Schindelhauer’s toolbox, i.e., important functions like masking, opening, and shuffling of cards and stacks, respectively. Some exotic operations are still missing, e.g., the possibility to insert a masked card secretly into a stack or the verifiable subset properties of stacks. All implemented operations are available for the original encoding scheme of Schindelhauer (see Section 2.2.1 [Data Types], page 11) and, of course, for the more efficient encoding scheme of Barnett and Smart (see Section 2.2.3.2 [BarnettSmartVTMF], page 29) as well.

**unsigned long int TMCG_SecurityLevel**

This read-only nonnegative integer represents the security parameter $\kappa$ which was given to the constructor of this class. It defines the number of sequential protocol iterations and hence the soundness error probability ($2^{-\kappa}$) of the zero-knowledge proofs in the encoding scheme of Schindelhauer. Further it defines the soundness error probability (also $2^{-\kappa}$) of the shuffle argument in the encoding scheme of Barnett and Smart, if the efficient protocols of Groth [Gr05, Gr10] and others [HSSV09] are not used.

**size_t TMCG_Players**

This read-only nonnegative integer represents the number of players as given to the constructor of this class.

**size_t TMCG_TypeBits**

This read-only nonnegative integer contains the number of bits that are necessary to encode the card types in the binary representation. It was given as an argument to the constructor of this class.
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SchindelhauerTMCG (const unsigned long security; const size_t k, const size_t w)

This constructor creates an instance, where security is a nonnegative integer that represents the security parameter \( \kappa \). The parameter \( k \) is the number of players and \( w \) is the number of bits which are necessary to represent all possible card types in a binary representation.

The integer \( \kappa \) controls the maximum soundness error probability \((2^{-\kappa})\) of the zero-knowledge proofs in the encoding scheme of Schindelhauer. Specifically, security defines the number of sequential iterations of the involved protocols and thus has a major impact on the computational and communication complexity. If the encoding scheme of Barnett and Smart [BS03] is used, then it only defines the soundness error probability (also \( 2^{-\kappa} \)) of the corresponding shuffle proof. However, if the efficient shuffle verification protocol of Groth [Gr05] is used, then the parameter security is dispensable, because the parameter \( \kappa \) given during instantiation of GrothVSSHE (e.g. the LibTMCG default security parameter \( \text{TMCG}_\text{GROTH}_L\_E \)) determines this soundness error probability \((2^{-\kappa})\). The similar holds for the verifiable rotation protocol [HSSV09], however, in this case there is no explicit security parameter for the soundness error.

Unfortunately, the parameters \( k \) and \( w \) have a major impact on the complexity in the encoding scheme of Schindelhauer, too. Therefore you should always use reasonable values here. For example, to create a deck with \( M \) different card types simply set \( w \) to \( \lceil \log_2 M \rceil \) which is a tight upper-bound for the applied binary representation. Furthermore, set \( k \) to the number of players which are really involved and not to a possible maximum value. Note that \( k \) and \( w \) are limited by the global constants \text{TMCG\_MAX\_PLAYERS} and \text{TMCG\_MAX\_TYPEBITS}, respectively.

void TMCG_CreateOpenCard (TMCG_Card& c, const TMCG_PublicKeyRing& ring, const size_t type)

This method initializes the open card \( c \) with the given type using the encoding scheme of Schindelhauer. The type MUST be an integer from the interval \([0, 2^w - 1]\), where \( w \) is the number given to the constructor of this class. The \( w \) MUST be the same number as used at creation of \( c \) (see Section 2.2.1 [Data Types], page 11). The parameter ring is a container with exactly \( k \) public keys, where \( k \) is the number given to the constructor of this class. The \( k \) MUST be the same number as used at the creation of \( c \).

void TMCG_CreateOpenCard (VTMF_Card& c, BarnettSmartVTMF_dlog* vtmf, const size_t type)

This method initializes the open card \( c \) with the given type using the encoding scheme of Barnett and Smart. The type MUST be an integer from the interval \([0, 2^w - 1]\), where \( w \) is the number given to the constructor of this class. The parameter vtmf is a pointer to an already initialized VTMF instance, i.e. the key generation protocol was successfully finished (see Section 2.2.3.2 [BarnettSmartVTMF], page 29, and \text{BarnettSmartVTMF\_dlog\_GroupQR}, respectively).

void TMCG_CreateCardSecret (TMCG_CardSecret& cs, const TMCG_PublicKeyRing& ring, const size_t index)

This method initializes the card secret \( cs \) with random values which is necessary to perform later a masking operation on a card. The parameter ring is a container with exactly \( k \) public keys, where \( k \) is the number given to the constructor of this class. It MUST be the same number as used at the creation of \( cs \) (see Section 2.2.1 [Data Types], page 11). The parameter index is from the interval \([0, k - 1]\) and determines the position of the players public key in the container ring.
void TMCG_CreateCardSecret (VTMF_CardSecret& cs, BarnettSmartVTMF_dlog* vtmf)

This method initializes the card secret cs with a random value which is necessary to perform later a masking operation on a card. The parameter vtmf is a pointer to an already initialized VTMF instance, i.e. the key generation protocol MUST be successfully finished (see Section 2.2.3.2 [BarnettSmartVTMF], page 29).

void TMCG_CreatePrivateCard (TMCG_Card& c, TMCG_CardSecret& cs, const TMCG_PublicKeyRing& ring, const size_t index, const size_t type)

This method initializes a masked card c with the given type and a corresponding card secret cs using the encoding scheme of Schindelhauer. The type MUST be an integer from the interval \([0, 2^w - 1]\), where \(w\) is the number given to the constructor of this class. The \(w\) MUST be the same number as used at creation of \(c\) and \(cs\) (see Section 2.2.1 [Data Types], page 11). The parameter ring is a container with exactly \(k\) public keys, where \(k\) is the number given to the constructor of this class. The \(k\) MUST be the same number as used at the creation of \(c\) and \(cs\). The parameter index is from the interval \([0, k - 1]\) and determines the position of the players public key in the container ring. Internally, TMCG_CreatePrivateCard calls
1. TMCG_CreateOpenCard to initialize \(c\) with type,
2. TMCG_CreateCardSecret to initialize \(cs\) with random values, and
3. TMCG_MaskCard to mask \(c\) with the secret \(cs\).

void TMCG_MaskCard (const TMCG_Card& c, TMCG_Card& cc, const TMCG_CardSecret& cs, const TMCG_PublicKeyRing& ring, const bool TimingAttackProtection = true)

This method performs a masking operation on the open or already masked card \(c\) using the encoding scheme of Schindelhauer. Finally it returns the result in \(cc\). The parameter cs MUST be an initialized fresh card secret which has NEVER been involved in a masking operation before. The parameters \(c\), \(cc\), and \(cs\) MUST be created such that their \(k\) and \(w\) corresponds to the numbers given to the constructor of this class, respectively. The parameter ring is a container with exactly \(k\) public keys. The protection against timing attacks is turned on, if TimingAttackProtection is set to true.

void TMCG_MaskCard (const VTMF_Card& c, VTMF_Card& cc, const VTMF_CardSecret& cs, BarnettSmartVTMF_dlog* vtmf, const bool TimingAttackProtection = true)

This method performs a masking operation on the open or already masked card \(c\) using the encoding scheme of Barnett and Smart. Finally it returns the result in \(cc\). Specifically, TMCG_MaskCard directly executes the masking operation of the verifiable masking protocol. The parameter cs MUST be an initialized fresh card secret which has NEVER been involved in a masking operation before. The parameter vtmf is a pointer to an already
initialized VTMF instance, i.e. the key generation protocol MUST be successfully finished (see Section 2.2.3.2 [BarnettSmartVTMF], page 29). The protection against timing attacks is turned on, if TimingAttackProtection is set to true.

void TMCG_ProveMaskCard (const TMCG_Card& c, const TMCG_Card& cc, const TMCG_CardSecret& cs, const TMCG_PublicKeyRing& ring, std::istream& in, std::ostream& out)

This method should be called by the prover after TMCG_MaskCard to show that he performed the masking operation correctly. The parameters c, cc, and cs are the input, the result, and the used card secret of TMCG_MaskCard, respectively. They MUST be created such that their k resp. w corresponds to the numbers given to the constructor of this class. The parameter ring is a container with exactly k public keys. The input/output protocol messages from and to the verifier are transmitted on the streams in and out, respectively.

void TMCG_ProveMaskCard (const VTMF_Card& c, const VTMF_Card& cc, const VTMF_CardSecret& cs, BarnettSmartVTMF_dlog* vtmf, std::istream& in, std::ostream& out)

This method should be executed by the prover after calling TMCG_MaskCard to show that he performed the masking operation correctly. Specifically, TMCG_ProveMaskCard directly calls the prove operation of the verifiable re-masking protocol. The parameters c, cc, and cs are the input, the result, and the used card secret of TMCG_MaskCard, respectively. The parameter vtmf is a pointer to an already initialized VTMF instance, i.e. the key generation protocol MUST be successfully finished. The input/output protocol messages from and to the verifier are transmitted on the streams in and out, respectively.

bool TMCG_VerifyMaskCard (const TMCG_Card& c, const TMCG_Card& cc, const TMCG_PublicKeyRing& ring, std::istream& in, std::ostream& out)

This method should be executed by the verifier to check whether or not a masking operation was performed correctly. The parameters c and cc are the input and the result of TMCG_MaskCard, respectively. They MUST be created such that their k resp. w corresponds to the numbers given to the constructor of this class. The parameter ring is a container with exactly k public keys. The input/output protocol messages from and to the prover are transmitted on the streams in and out, respectively. The method returns true, if everything was sound.

bool TMCG_VerifyMaskCard (const VTMF_Card& c, const VTMF_Card& cc, BarnettSmartVTMF_dlog* vtmf, std::istream& in, std::ostream& out)

This method should be executed by the verifier to check whether or not a masking operation was performed correctly. Specifically, TMCG_VerifyMaskCard directly calls the verify operation of the verifiable re-masking protocol. The parameters c and cc are the input and the result of TMCG_MaskCard, respectively. The parameter vtmf is a pointer to an already initialized VTMF instance, i.e. the key generation protocol MUST be successfully finished. The input/output protocol messages from and to the prover are transmitted on the streams in and out, respectively. The method returns true, if everything was sound.

void TMCG_ProveCardSecret (const TMCG_Card& c, const TMCG_SecretKey& key, const size_t index, std::istream& in, std::ostream& out)

This method is used to reveal the card type of c to a verifier. Every player must execute this method as prover. The card c MUST be created such that its k resp. w corresponds to the numbers given to the constructor of this class. The parameter key is the corresponding secret key (see Section 2.2.1 [Data Types], page 11) of the prover. The parameter
index is from the interval \([0, k - 1]\) and contains the position of the provers public key in the container ring (same as in \texttt{TMCG\_CreateCardSecret}). The input/output protocol messages from and to the verifier are transmitted on the streams \textit{in} and \textit{out}, respectively.

\begin{verbatim}
void TMCG\_ProveCardSecret (const VTMF\_Card& c, BarnettSmartVTMF\_dlog* vtmf, std::istream& in, std::ostream& out)
This method is used to reveal the card type of \(c\) to a verifier. Every player must execute this method as prover. Specifically, \texttt{TMCG\_ProveCardSecret} directly calls the prove operation of the verifiable decryption protocol. The parameter \(vtmf\) is a pointer to an already initialized VTMF instance, i.e. the key generation protocol MUST be successfully finished. The input/output protocol messages from and to the verifier are transmitted on the streams \textit{in} and \textit{out}, respectively.

bool TMCG\_VerifyCardSecret (const TMCG\_Card& c, TMCG\_CardSecret& cs, const TMCG\_PublicKey& key, const size_t index, std::istream& in, std::ostream& out)
This method is used to verify and accumulate card type information regarding \(c\) that are supplied by a prover. It is the opposite method of \texttt{TMCG\_ProveCardSecret} and must be executed by the player who wants to know the type. The secrets provided by the single provers are accumulated in the parameter \(cs\). Thus \(c\) and \(cs\) MUST be created such that their \(k\) resp. \(w\) corresponds to the numbers given to the constructor of this class. The parameter \(key\) is the corresponding public key (see Section 2.2.1 [Data Types], page 11) of the prover. The parameter \(index\) is from the interval \([0, k - 1]\) and contains the position of the provers public key in the container ring (same as in \texttt{TMCG\_CreateCardSecret}). The input/output protocol messages from and to the prover are transmitted on the streams \textit{in} and \textit{out}, respectively.

bool TMCG\_VerifyCardSecret (const VTMF\_Card& c, BarnettSmartVTMF\_dlog* vtmf, std::istream& in, std::ostream& out)
This method is used to verify and accumulate card type information regarding \(c\) that are supplied by a prover. It is the opposite method of \texttt{TMCG\_ProveCardSecret} and must be executed by the player who wants to know the type of \(c\). The information is accumulated in the parameter \(cs\). Thus \(c\) and \(cs\) MUST be created such that their \(k\) resp. \(w\) corresponds to the numbers given to the constructor of this class. The parameter \(key\) is the corresponding secret key (see Section 2.2.1 [Data Types], page 11) of the player. The parameter \(index\) is from the interval \([0, k - 1]\) and contains the position of the players public key in the container ring (same as in \texttt{TMCG\_CreateCardSecret}).

void TMCG\_SelfCardSecret (const TMCG\_Card& c, TMCG\_CardSecret& cs, const TMCG\_SecretKey& key, const size_t index)
This method is used to compute and accumulate card type information regarding \(c\). Analogously to \texttt{TMCG\_VerifyCardSecret} it must be executed by the player who wants to know the type of \(c\). The information is accumulated in the parameter \(cs\). Thus \(c\) and \(cs\) MUST be created such that their \(k\) resp. \(w\) corresponds to the numbers given to the constructor of this class. The parameter \(key\) is the corresponding secret key (see Section 2.2.1 [Data Types], page 11) of the player. The parameter \(index\) is from the interval \([0, k - 1]\) and contains the position of the players public key in the container ring (same as in \texttt{TMCG\_CreateCardSecret}).
\end{verbatim}
void TMCG_SelfCardSecret (const VTMF_Card& c, BarnettSmartVTMF_dlog* vtmf)  
This method is used to compute and accumulate card type information regarding c. It MUST be called by the player who wants to know the type of c BEFORE TMCG_VerifyCardSecret and TMCG_TypeOfCard are executed. The secrets provided by the player are accumulated internally, thus this method cannot be interleaved with the opening of other cards. Specifically, TMCG_SelfCardSecret directly calls the initialize operation of the verifiable decryption protocol. The parameter vtmf is a pointer to an already initialized VTMF instance, i.e. the key generation protocol MUST be successfully finished.

size_t TMCG_TypeOfCard (const TMCG_CardSecret& cs)  
This method returns the type of a masked card provided that the type information were properly accumulated in cs before (by calling TMCG_SelfCardSecret and TMCG_VerifyCardSecret, respectively).

size_t TMCG_TypeOfCard (const VTMF_Card& c, BarnettSmartVTMF_dlog* vtmf)  
This method returns the type of a masked card c provided that the type information regarding c were properly accumulated internally before (by calling TMCG_SelfCardSecret and TMCG_VerifyCardSecret, respectively). It returns the value TMCG_MaxCardType, if the opening operation failed or if the card type was not among the set of valid types. This method MUST be performed by the player who wants to know the type AFTER TMCG_SelfCardSecret and TMCG_VerifyCardSecret are executed. Specifically, TMCG_TypeOfCard directly calls the finalize operation of the verifiable decryption protocol. The parameter vtmf is a pointer to an already initialized VTMF instance, i.e. the key generation protocol MUST be successfully finished.

size_t TMCG_CreateStackSecret (TMCG_StackSecret<TMCG_CardSecret>& ss, const bool cyclic, const TMCG_PublicKeyRing& ring, const size_t index, const size_t size)  
This method initializes the stack secret ss with a randomly and uniformly chosen permutation (using the algorithm of Knuth) and fresh card secrets. Later this stack secret can be used to perform a secret shuffle operation on a stack. If the parameter cyclic is set to true, then the permutation is only a cyclic shift which might be of interest for particular operations, e.g. cutting the deck. The parameter ring is a container with exactly k public keys, where k is the number given to the constructor of this class. The parameter index is from the interval [0, k − 1] and contains the position of the players public key in the container ring. The parameter size determines the size of the created stack secret, i.e. the number of cards in the corresponding stack. The size is upper-bounded by TMCG_MAX_CARDS. The method returns the offset of the cyclic shift, if cyclic was set to true. Otherwise, the value 0 is returned.

size_t TMCG_CreateStackSecret (TMCG_StackSecret<VTMF_CardSecret>& ss, const bool cyclic, const size_t size, BarnettSmartVTMF_dlog* vtmf)  
This method initializes the stack secret ss with a randomly and uniformly chosen permutation (using the algorithm of Knuth) and fresh card secrets. Later this stack secret can be used to perform a secret shuffle operation on a stack. If the parameter cyclic is set to true, then the permutation is only a cyclic shift which might be of interest for particular operations, e.g. cutting the deck. The parameter size determines the size of the created stack secret, i.e. the number of cards in the corresponding stack. The size is upper-bounded by TMCG_MAX_CARDS. The parameter vtmf is a pointer to an already initialized VTMF instance, i.e. the key generation protocol MUST be successfully finished.
The method returns the offset of the cyclic shift, if cyclic was set to true. Otherwise, the value 0 is returned.

```c++
void TMCG_CreateStackSecret
    (TMCG_StackSecret<TMCG_CardSecret>& ss, const
    std::vector<size_t>& pi, const TMCG_PublicKeyRing& ring, const
    size_t index, const size_t size)
```

This method initializes the stack secret ss with a given permutation pi and fresh card secrets. Later this stack secret can be used to perform a secret shuffle operation on a stack. The parameter ring is a container with exactly k public keys, where k is the number given to the constructor of this class. The parameter index is from the interval [0, k – 1] and contains the position of the players public key in the container ring. The parameter size determines the size of the created stack secret, i.e. the number of cards in the corresponding stack. The size is upper-bounded by TMCG_MAX_CARDS.

```c++
void TMCG_CreateStackSecret
    (TMCG_StackSecret<VTMF_CardSecret>& ss, const
    std::vector<size_t>& pi, const size_t size, BarnettSmartVTMF_dlog* vtmf)
```

This method initializes the stack secret ss with a given permutation pi and fresh card secrets. Later this stack secret can be used to perform a secret shuffle operation on a stack. The parameter size determines the size of the created stack secret, i.e. the number of cards in the corresponding stack. The size is upper-bounded by TMCG_MAX_CARDS. The parameter vtmf is a pointer to an already initialized VTMF instance, i.e. the key generation protocol MUST be successfully finished.

```c++
void TMCG_MixStack (const
    TMCG_Stack<TMCG_Card>& s, TMCG_Stack<TMCG_Card>& s2, const
    TMCG_StackSecret<TMCG_CardSecret>& ss, const TMCG_PublicKeyRing&
    ring, const bool TimingAttackProtection =true)
```

This method shuffles a given stack s according to the previously created stack secret ss (see Section 2.2.1 [Data Types], page 11). The result of the shuffle is returned in s2. The parameter ss MUST be a fresh stack secret which has NEVER been involved in a shuffle operation before. The parameters s and ss MUST be of the same size. The parameter ring is a container with exactly k public keys, where k is the number given to the constructor of this class. The protection against timing attacks is turned on, if TimingAttackProtection is set to true.

```c++
void TMCG_MixStack (const
    TMCG_Stack<VTMF_Card>& s, TMCG_Stack<VTMF_Card>& s2, const
    TMCG_StackSecret<VTMF_CardSecret>& ss, BarnettSmartVTMF_dlog* vtmf, const bool TimingAttackProtection =true)
```

This method shuffles a given stack s according to the previously created stack secret ss (see Section 2.2.1 [Data Types], page 11). The result of the shuffle is returned in s2. The parameter ss MUST be a fresh stack secret which has NEVER been involved in a shuffle operation before. The parameters s and ss MUST be of the same size. The parameter vtmf is a pointer to an already initialized VTMF instance, i.e. the key generation protocol MUST be successfully finished. The protection against timing attacks is turned on, if TimingAttackProtection is set to true.
void TMCG_ProveStackEquality (const TMCG_Stack<TMCG_Card>& s, const TMCG_Stack<TMCG_Card>& s2, const TMCG_StackSecret<TMCG_CardSecret>& ss, const bool cyclic, const TMCG_PublicKeyRing& ring, const size_t index, std::istream& in, std::ostream& out)

This method should be called by the prover after TMCG_MixStack to show that he performed the shuffle operation correctly. The parameters s, s2, and ss are the input, the result, and the used stack secret of TMCG_MixStack, respectively. Of course, the parameters s, s2, and ss MUST be of the same size. The parameter cyclic determines whether a cyclic shift or a full permutation was used to shuffle the stack. The parameter ring is a container with exactly k public keys, where k is the number given to the constructor of this class. The parameter index is from the interval [0, k − 1] and contains the position of the provers public key in the container ring. The input/output protocol messages from and to the verifier are transmitted on the streams in and out, respectively.

void TMCG_ProveStackEquality (const TMCG_Stack<VTMF_Card>& s, const TMCG_Stack<VTMF_Card>& s2, const TMCG_StackSecret<VTMF_CardSecret>& ss, const bool cyclic, BarnettSmartVTMF_dlog* vtmf, std::istream& in, std::ostream& out)

This method should be called by the prover after TMCG_MixStack to show that he performed the shuffle operation correctly. The parameters s, s2, and ss are the input, the result, and the used stack secret of TMCG_MixStack, respectively. Of course, the parameters s, s2, and ss MUST be of the same size. The parameter cyclic determines whether a cyclic shift or a full permutation was used to shuffle the stack. The parameter vtmf is a pointer to an already initialized VTMF instance, i.e. the key generation protocol MUST be successfully finished. The input/output protocol messages from and to the verifier are transmitted on the streams in and out, respectively.

void TMCG_ProveStackEquality_Groth (const TMCG_Stack<VTMF_Card>& s, const TMCG_Stack<VTMF_Card>& s2, const TMCG_StackSecret<VTMF_CardSecret>& ss, BarnettSmartVTMF_dlog* vtmf, GrothVSSHE* vsshe, std::istream& in, std::ostream& out)

This is a method like above. The only difference is that the more efficient interactive shuffle verification protocol of Groth [Gr05] is used. Thus vsshe is a pointer to a proper initialized instance of GrothVSSHE. The rest of the arguments are the same.

void TMCG_ProveStackEquality_Groth_noninteractive (const TMCG_Stack<VTMF_Card>& s, const TMCG_Stack<VTMF_Card>& s2, const TMCG_StackSecret<VTMF_CardSecret>& ss, BarnettSmartVTMF_dlog* vtmf, GrothVSSHE* vsshe, std::ostream& out)

This is a method like above. The difference is that the non-interactive version of the shuffle verification protocol is used. Thus only an output stream out is given, for example std::stringstream can be appropriate here. Again vsshe is a pointer to a proper initialized instance of GrothVSSHE. The rest of the arguments are the same.

void TMCG_ProveStackEquality_Hoogh (const TMCG_Stack<VTMF_Card>& s, const TMCG_Stack<VTMF_Card>& s2, const TMCG_StackSecret<VTMF_CardSecret>& ss, BarnettSmartVTMF_dlog* vtmf, HooghSchoenmakersSkoricVillegasVRHE* vrhe, std::istream& in, std::ostream& out)

This is a method like above. The only difference is that the more efficient rotation verification protocol [HSSV09] is used. Thus vrhe is a pointer to an initialized instance of HooghSchoenmakersSkoricVillegasVRHE. The rest of the arguments are the same.
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```cpp
void TMCG_ProveStackEquality_Hoogh_noninteractive (const TMCG_Stack<VTMF_Card>& s, const TMCG_Stack<VTMF_Card>& s2, const TMCG_StackSecret<VTMF_CardSecret>& ss, BarnettSmartVTMF_dlog* vtmf, HooghSchoenmakersSkoricVillegasVRHE* vrhe, std::ostream& out)
```

This is a method like above. The difference is that the non-interactive version of the rotation verification protocol is used. Thus only an output stream `out` is given, for example `std::stringstream` can be appropriate here. Again `vrhe` is a pointer to an initialized instance of HooghSchoenmakersSkoricVillegasVRHE. The rest of the arguments are the same.

```cpp
bool TMCG_VerifyStackEquality (const TMCG_Stack<TMCG_Card>& s, const TMCG_Stack<TMCG_Card>& s2, const bool cyclic, const TMCG_PublicKeyRing& ring, std::istream& in, std::ostream& out)
```

This method should be executed by the verifier to check whether or not a shuffle operation was performed correctly. The parameters `s` and `s2` are the input and the result of `TMCG_MixStack`, respectively. Of course, the parameters `s` and `s2` should be of the same size. The parameter `cyclic` determines whether a cyclic shift or a full permutation was used to shuffle the stack. The parameter `ring` is a container with exactly `k` public keys, where `k` is the number given to the constructor of this class. The input/output protocol messages from and to the prover are transmitted on the streams `in` and `out`, respectively. This method returns `true`, if the shuffle operation was successfully verified.

```cpp
bool TMCG_VerifyStackEquality_Groth (const TMCG_Stack<VTMF_Card>& s, const TMCG_Stack<VTMF_Card>& s2, BarnettSmartVTMF_dlog* vtmf, GrothVSSHE* vsshe, std::istream& in, std::ostream& out)
```

This is a method like above. The only difference is that the more efficient shuffle verification protocol of Groth is used. Thus `vsshe` is a pointer to an already initialized VTMF instance, i.e. the key generation protocol MUST be successfully finished. The input/output protocol messages from and to the verifier are transmitted on the streams `in` and `out`, respectively. This method returns `true`, if the shuffle operation was successfully verified.

```cpp
bool TMCG_VerifyStackEquality_Groth_noninteractive (const TMCG_Stack<VTMF_Card>& s, const TMCG_Stack<VTMF_Card>& s2, BarnettSmartVTMF_dlog* vtmf, GrothVSSHE* vsshe, std::istream& in)
```

This is a method like above. The difference is that the non-interactive version of the shuffle verification protocol is used. Thus only an input stream `in` is given, for example `std::stringstream` can be appropriate here. Again `vsshe` is a pointer to an initialized instance of GrothVSSHE. The rest of the arguments and the returned values are the same.
bool TMCG_VerifyStackEquality_Hoogh (const
TMCG_Stack<VTMF_Card>& s, const TMCG_Stack<VTMF_Card>& s2,
BarnettSmartVTMF_dlog* vtmf,
HooghSchoenmakersSkoricVillegasVRHE* vrhe, std::istream& in,
std::ostream& out)
This is a method like above. The only difference is that the more efficient rotation verification protocol [HSSV09] is used. Thus vrhe is a pointer to an initialized instance of HooghSchoenmakersSkoricVillegasVRHE. The rest of the arguments and the returned values are the same.

bool
TMCG_VerifyStackEquality_Hoogh_noninteractive (const
TMCG_Stack<VTMF_Card>& s, const TMCG_Stack<VTMF_Card>& s2,
BarnettSmartVTMF_dlog* vtmf,
HooghSchoenmakersSkoricVillegasVRHE* vrhe, std::istream& in)
This is a method like above. The difference is that the non-interactive version of the rotation verification protocol is used. Thus only an input stream in is given, for example std::stringstream can be appropriate here. Again vrhe is a pointer to an initialized instance of HooghSchoenmakersSkoricVillegasVRHE. The rest of the arguments and the returned values are the same.

~SchindelhauerTMCG ()
This destructor releases all occupied resources.
3 Examples

The following examples explain most of the steps that are necessary to create a secure and verifiable card game with LibTMCG. We consider an application with five permanent players (denoted by $P_0$, $P_1$, $P_2$, $P_3$, and $P_4$) and a regular deck of 52 different cards. For convenience only the more efficient card encoding scheme of Barnett and Smart [BS03] is described. Additionally, we complete our exposition with code fragments which show the application of the fast shuffle verification protocol due to Groth [Gr05, Gr10] with an interactive or even non-interactive instantiation of the zero-knowledge proofs. On modern computers this approach achieves good real world performance and simultaneously keeps the cheating probability negligible.

Throughout the remaining pages we assume that all players are pairwise connected by authenticated communication channels. These channels are organized in input resp. output streams, where `input_stream[i]` resp. `output_stream[i]` denote the corresponding `std::istream` resp. `std::ostream` instance for the communication with player $P_i$.

1. **Library Initialization**

The very first step that should be done is the initialization of LibTMCG. You can simply perform this task by calling the function `init_libTMCG` and evaluating the return code.

```cpp
if (!init_libTMCG())
    std::cerr << "Initialization of LibTMCG failed!" << std::endl;
```

Additionally, in most cases it is useful to check the installed version of the library by comparing the desired value with the returned string of the function `version_libTMCG`.

2. **Setup Communication Channels**

Some multiparty protocols require additional asynchronous point-to-point communication channels (authenticated and private) and a reliable broadcast channel. The following example shows, how to setup these channels for player $P_i$:

```cpp
// create asynchronous private unicast channels
aiounicast_select *aiou = new aiounicast_select(5, i, uP_in, uP_out, uP_key,
    aiounicast::aio_scheduler_roundrobin, aiounicast::aio_timeout_short);

// create asynchronous private broadcast channels
aiounicast_select *aiou2 = new aiounicast_select(5, i, bP_in, bP_out, bP_key,
    aiounicast::aio_scheduler_roundrobin, aiounicast::aio_timeout_short);

// create an instance of a reliable broadcast protocol (RBC)
std::string myID = "example-poker-libTMCG-reference-manual";
CachinKursavePetzoldShoupRBC *rbc = new CachinKursavePetzoldShoupRBC(5, 1, i,
    aiou2, aiounicast::aio_scheduler_roundrobin, aiounicast::aio_timeout_short);
    rbc->setID(myID);
```

We assume that pairwise private keys (e.g. passphrases) have been exchanged (i.e. vector `uP_key` resp. `bP_key`) and point-to-point links (i.e. input file descriptors in vector `uP_in` resp. `bP_in` and output file descriptors in vector `uP_out` resp. `bP_out`) have been already established.

1. We assume that the players are ordered in a natural way such that we can use an uniform nomenclature.
3.3 Session Initialization and Key Generation

In the next step we create an instance of the class `SchindelhauerTMCG`. The first parameter determines the number of protocol iterations $\kappa$ which upper-bounds the cheating probability by $2^{-\kappa}$. In our example the used value 64 defines a maximum cheating probability of $5.421010862 \cdot 10^{-20}$ which is reasonable small for our purposes.\footnote{If we use the encoding scheme of Barnett and Smart and only Groth’s shuffle protocol during the game, then the error probability is even smaller, because the security parameters of them are fixed within LibTMCG (see Section 2.1 [Preprocessor Defined Global Symbols], page 8).} The second parameter passes the number of players to the instance which is simply 5 in our case. The last argument defines the number of bits that are necessary to encode all card types in a binary representation. The given value 6 allows the encoding of $2^6 = 64$ different card types at maximum. This is enough to form our deck of 52 cards.

```c
SchindelhauerTMCG *tmcg = new SchindelhauerTMCG(64, 5, 6);
```

In our example we would like to use the more efficient encoding scheme of Barnett and Smart, thus we create an instance of `BarnettSmartVTMF_dlog`. However, a particular player has to act as a leader who performs the generation of the group $G$ as a common reference. In our case $P_0$ will be the session leader. First, he executes the constructor of the class `BarnettSmartVTMF_dlog` that may take some time.

```c
BarnettSmartVTMF_dlog *vtmf = new BarnettSmartVTMF_dlog();
```

Afterwards he checks the generated group $G$ and sends the public parameters to all other players (their corresponding stream indices are 1, 2, 3, and 4, respectively).

```c
if (!vtmf->CheckGroup())
    std::cerr << "Group G was not correctly generated!" << std::endl;
for (size_t i = 1; i < 5; i++)
    vtmf->PublishGroup(output_stream[i]);
```

The other players receive the group parameters from $P_0$ and use them to initialize their corresponding instances of `BarnettSmartVTMF_dlog`. It is very important that they also check, whether the group $G$ was correctly generated by the leader.

```c
BarnettSmartVTMF_dlog *vtmf =
    new BarnettSmartVTMF_dlog(input_stream[0]);
if (!vtmf->CheckGroup())
    std::cerr << "Group G was not correctly generated!" << std::endl;
```

Afterwards the key generation protocol is carried out. First, every player generates his own VTMF key. The private key material is stored internally and will never be exposed.

```c
vtmf->KeyGenerationProtocol_GenerateKey();
```

Then every player $P_j$ sends the public part of his VTMF key along with a non-interactive zero-knowledge proof of knowledge (NIZK) to each other player. The appended proof shows that he indeed knows the corresponding secret key. However, due to the non-interactive nature of this proof we have to be careful, if the same group $G$ is eventually used again. It is even better to generate a fresh group (common reference) and key for each new game session.
for (size_t i = 0; i < 5; i++)
{
    if (i != j)
        vtmf->KeyGenerationProtocol_PublishKey(output_stream[i]);
}

After sending, $P_j$ receives the public keys of the other players. Of course she checks, whether these keys are correctly generated, and she updates the common public key $h$.

for (size_t i = 0; i < 5; i++)
{
    if (i != j)
    {
        if (!vtmf->KeyGenerationProtocol_UpdateKey(input_stream[i]))
            std::cerr << "Public key was not correctly generated!" << std::endl;
    }
}

Finally, every player must finalize the key generation protocol.

vtmf->KeyGenerationProtocol_Finalize();

For some sophisticated parts of LibTMCG a distributed coin flipping protocol is necessary. It protects the honest-verifier zero-knowledge proofs or arguments against malicious verifiers. So, all players should execute as an initialization procedure:

JareckiLysyanskayaEDCF *edcf;
edcf = new JareckiLysyanskayaEDCF(5, 5, vtmf->p, vtmf->q, vtmf->g, vtmf->h);
if (!edcf->CheckGroup())
    std::cerr << "Group $G$ was not correctly generated!" << std::endl;

If we want to use the more efficient shuffle verification protocol of Groth, then $P_0$ must also create an instance of GrothVSSHE. The first argument determines the maximum stack size of which the correctness of a shuffle will be proven. The other parameters are obtained from the former created VTMF instance $vtmf$. It is important that the key generation protocol has been finalized before the common public key $h$ (i.e. $vtmf->h$) is passed, because this value is checked within.

GrothVSSHE *vsshe = new GrothVSSHE(52, vtmf->p, vtmf->q, vtmf->k,
vtmf->g, vtmf->h);

Again, $P_0$ will send the public parameters of the VSSHE instance to all other players.

for (size_t i = 1; i < 5; i++)
vsshe->PublishGroup(output_stream[i]);

The other players receive these parameters from the leader and use them to initialize their corresponding instances of GrothVSSHE. Again, it is important to check, whether the parameters were correctly chosen by the leader.
Last but not least the setup of some internal generators must be accomplished by all players in a verifiable way (see Section 2.2.3.4 [GrothVSSHE], page 34).\footnote{There is also the possibility to use the simple variant of SetupGenerators_publiccoin with the already generated public key $h$ as a common random value. However, this value should be refreshed periodically.}

### 3.4 Operations on Cards

Now we are ready to perform several operations on cards. We start with some basic stuff which might be of interest in particular situations. However, it is often more convenient to work directly with stacks, as explained later.

#### 3.4.1 Creating an Open Card

The creation of an open card is very simple. The following code creates a card of type 7.

```cpp
VTMF_Card c;
tmcg->TMCG_CreateOpenCard(c, vtmf, 7);
```

#### 3.4.2 Masking and Re-masking of a Card

Now the previously created card $c$ will be masked to hide its type. Then $cc$ is sent to $P_1$.

```cpp
VTMF_Card cc;
VTMF_CardSecret cs;
tmcg->TMCG_CreateCardSecret(cs, vtmf);
tmcg->TMCG_MaskCard(c, cc, cs, vtmf);
out_stream[1] << cc << std::endl;
```

$P_1$ receives the card $cc$, re-masks them, and sends the result $ccc$ back to the player $P_0$. Further he proves that the masking operation was performed correctly.
VTMF_Card cc, ccc;
VTMF_CardSecret ccs;
in_stream[0] >> cc;
if (!in_stream[0].good())
    std::cerr << "Read or parse error!" << std::endl;
tmcg->TMCG_CreateCardSecret(ccs, vtmf);
tmcg->TMCG_MaskCard(cc, ccc, ccs, vtmf);
out_stream[0] << ccc << std::endl;
tmcg->TMCG_ProveMaskCard(cc, ccc, ccs, vtmf, in_stream[0], out_stream[0]);

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3.5.1 Creating the Deck

A quite common operation is the creation of a card deck. The deck will initially be represented by an open stack (see TMCG_OpenStack) called deck. Every player creates his own instance of the deck, which consists of 52 open cards of different type in our example.

```cpp
TMCG_OpenStack<VTMF_Card> deck;
for (size_t type = 0; type < 52; type++)
{
    VTMF_Card c;
    tmcg->TMCG_CreateOpenCard(c, vtmf, type);
    deck.push(type, c);
}
```

Note that the instances of the deck must be consistent for all players, that means, the order of the open cards and their types must be exactly the same for all players.

Finally, we copy the deck to a regular stack s for further processing:

```cpp
TMCG_Stack<VTMF_Card> s;
s.push(deck);
```

3.5.2 Shuffling the Deck

Every player must perform a shuffle of the deck, because only such a procedure guarantees that no coalition has influence on the outcome. Thus we build a shuffle chain (e.g. using the strict total order $P_i < P_j$, if and only if $i < j$) such that each player shuffles the deck once.

First the regular stack s is initialized with open cards from deck. Then each player shuffles the stack (see Section 2.2.3.6 [SchindelhauerTMCG], page 37, i.e, TMCG_MixStack) using randomness (see TMCG_CreateStackSecret) and proves the correctness of this operation (see TMCG_ProveStackEquality). Consequently, every player should verify these proofs (see TMCG_VerifyStackEquality) and complain deviations immediately. Finally, the stack s contains the shuffled result. Consider the following code fragment for the player $P_j$. 
for (size_t i = 0; i < 5; i++)
{
    TMCG_Stack<VTMF_Card> s2;
    if (i == j)
    {
        TMCG_StackSecret<VTMF_CardSecret> ss;
        tmcg->TMCG_CreateStackSecret(ss, false, s.size(), vtmf);
        tmcg->TMCG_MixStack(s, s2, ss, vtmf);
        for (size_t i2 = 0; i2 < 5; i2++)
        {
            if (i2 == j)
                continue;
            out_stream[i2] << s2 << std::endl;
            tmcg->TMCG_ProveStackEquality(s, s2, ss, false, vtmf,
                in_stream[i2], out_stream[i2]);
        }
    }
    in_stream[i] >> s2;
    if (!in_stream[i].good())
        std::cerr << "Read or parse error!" << std::endl;
    if (!tmcg->TMCG_VerifyStackEquality(s, s2, false, vtmf,
        in_stream[i], out_stream[i]))
        std::cerr << "Verification failed!" << std::endl;
    s = s2;
}

If you want to use the more efficient shuffle verification protocol of Groth, then you have to replace TMCG_ProveStackEquality and TMCG_VerifyStackEquality by TMCG_ProveStackEquality_Groth and TMCG_VerifyStackEquality_Groth, respectively.4

3.5.3 Drawing a Card from the Deck
Now every player has the same shuffled deck s and nobody knows in which order the 52 cards are stacked. Therefore you can simply use any drawing strategy to obtain a players hand. For example, look at the following code that draws two cards from s for each player.

TMCG_Stack<VTMF_Card> hand[5];
for (size_t i = 0; i < 5; i++)
{
    VTMF_Card c1, c2;
    s.pop(c1), s.pop(c2);
    hand[i].push(c1), hand[i].push(c2);
}

4 The non-interactive version of Groth’s protocol (see TMCG_ProveStackEquality_Groth_noninteractive and TMCG_VerifyStackEquality_Groth_noninteractive) provides an even more efficient implementation, because the prover has to compute the argument of correctness only once. Additionally, it protects against malicious verifiers and reduces the communication complexity, i.e. instead of O(n^2) the prover must perform only O(n) steps. Thus this approach is strongly recommended. However, the security then relies on the random oracle assumption. Please have a look at the included source code tests/t-poker-noninteractive.cc to get a clue.
Further, probably you want disclose the card types to the corresponding player. Consider the code fragment for the player $P_j$: Every player receives the necessary information from the other players and she computes the card types of her hand $\text{hand}[j]$. Finally, these types are stored together with the masked cards in the open stack $\text{private_hand}$.

```cpp
TMCG_OpenStack<VTMF_Card> private_hand;
for (size_t i = 0; i < 5; i++)
{
    if (i == j)
    {
        for (size_t k = 0; k < hand[j].size(); k++)
        {
            tmcg->TMCG_SelfCardSecret(hand[j][k], vtmf);
            for (size_t i2 = 0; i2 < 5; i2++)
            {
                if (i2 == j)
                    continue;
                if (!tmcg->TMCG_VerifyCardSecret(hand[j][k], vtmf,
                    in_stream[i2], out_stream[i2]))
                    std::cerr << "Verification failed!" << std::endl;
            }
            private_hand.push(tmcg->TMCG_TypeOfCard(hand[j][k], vtmf),
                hand[j][k]);
        }
    }
    else
    {
        for (size_t k = 0; k < hand[i].size(); k++)
        {
            tmcg->TMCG_ProveCardSecret(hand[i][k], vtmf,
                in_stream[i], out_stream[i]);
        }
    }
}
```

The example can be modified in a straightforward way to publicly disclose a card from a players hand or from the remaining stack $s$, i.e. to lay down the card face-up on the table.

### 3.6 Quit a Session
In the last step you should release all occupied resources.

```cpp
delete vsshe, delete edcf, delete vtmf, delete tmcg;
delete rbc, delete aiou2, delete aiou;
```
4 Tools

LibTMCG provides some additional protocols that may be of independent interest.

4.1 Distributed Key Generation and Threshold Cryptography

We have implemented a robust and secure protocol for Distributed Key Generation (DKG) of public-key cryptosystems (see Rosario Gennaro, Stanislaw Jarecki, Hugo Krawczyk, and Tal Rabin: Secure Distributed Key Generation for Discrete-Log Based Cryptosystems, Journal of Cryptology, Vol. 20 Nr. 1, Springer 2007). Moreover, LibTMCG also provides a robust and secure protocol for threshold DSA/DSS (see Ran Canetti, Rosario Gennaro, Stanislaw Jarecki, Hugo Krawczyk, and Tal Rabin: Adaptive Security for Threshold Cryptosystems, Advances in Cryptology – Proceedings of CRYPTO ’99, Lecture Notes in Computer Science 1666, Springer 1999). Robustness and security means that up to \( t \leq n/2 \) resp. \( t \leq n/3 \) parties can act maliciously and the protocols still produce some result (e.g. a valid DSA/DSS signature on a given hash value).

The current implementation is in experimental state and should not be used in production environments. Motivation, cryptographical background and some usage scenarios have been presented recently at 26th Krypto-Tag (GI Working Group) and Datengarten/81 (CCCB). Please consult the slides for a first overview. The former DKG tools have been removed from this release. These programs are continued in a separate package called Distributed Privacy Guard (DKGPG).

Please report any bugs to the maintainer of LibTMCG. Every help with development or testing of these DKG protocols and programs is very welcome!
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